



# Management Science

## Analytic Hierarchy Process

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# Analytic Hierarchy Process (AHP)

- This is a course about quantitative methods
- However, it is often difficult to conceptualize or quantify all the different elements of a problem
- The AHP was formulated to counter those situations, and is a mathematically-based theory
- It employs two key aspects:
  - (1) data from the various variables that make up the decision
  - (2) judgments about those variables



- How do you determine the best route to school/work each day ?



■ Do not look ahead !



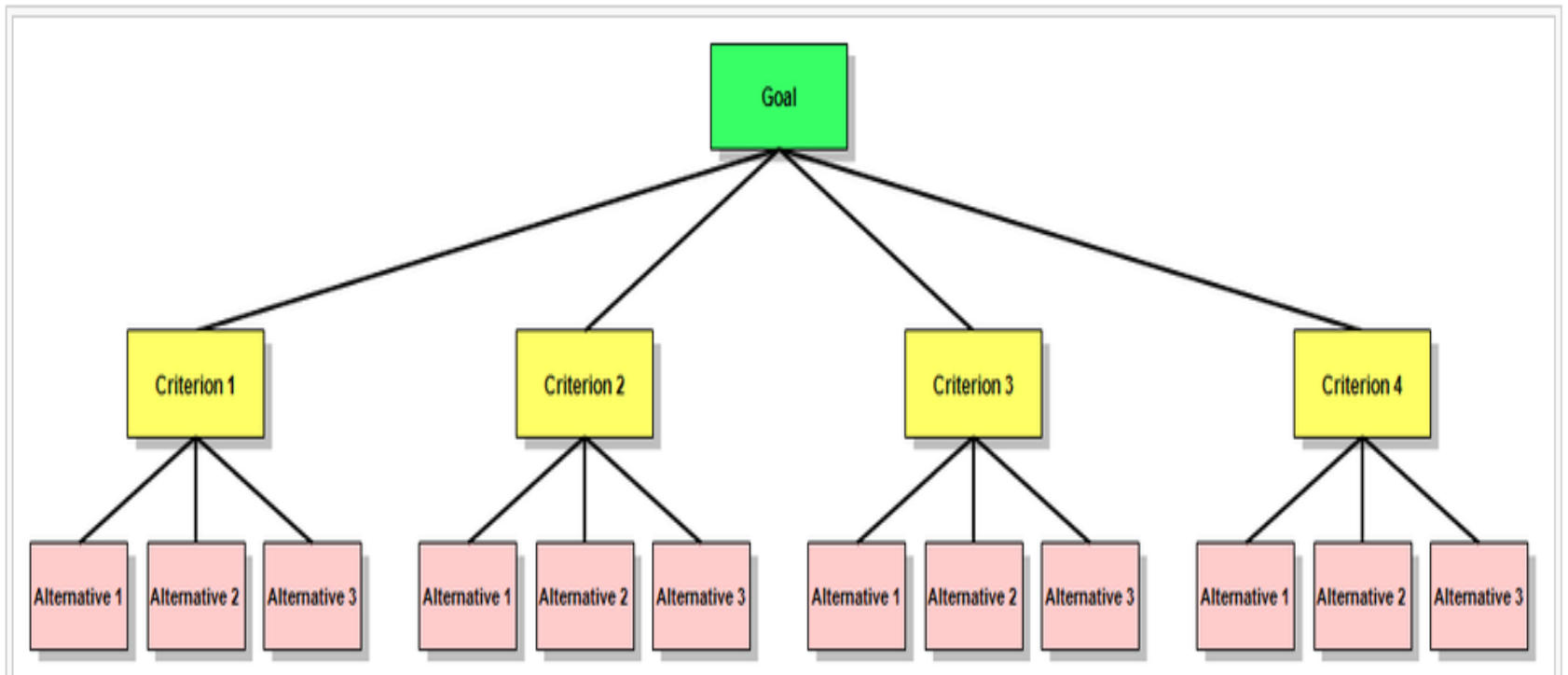
- The fastest route ?
- The cheapest route ?
- The safest route ?
- The most scenic route ?
- The route with the most coffee drive thru's ?



# Analytic Hierarchy Process (con't)

- The AHP requires taking the following steps:
  1. Structuring the decision into a hierarchical model
  2. Pairwise comparison of all objects and alternative solutions
- The form of the model has four elements:
  1. *Goal* – the desired outcome
  2. *Criteria* – elements that comprise the goal (objectives)
  3. *Subcriteria* – elements inside the criteria
  4. *Alternatives* – solutions or choices available
- This format allows decision makers to examine every part of a complex problem

# Analytic Hierarchy Process (con't)



**AHP will evaluate each alternative for each criteria.**

# AHP Example

**Goal:**

**Choose a Leader**

**Criteria:**

**Age**

**Experience**

**Education**

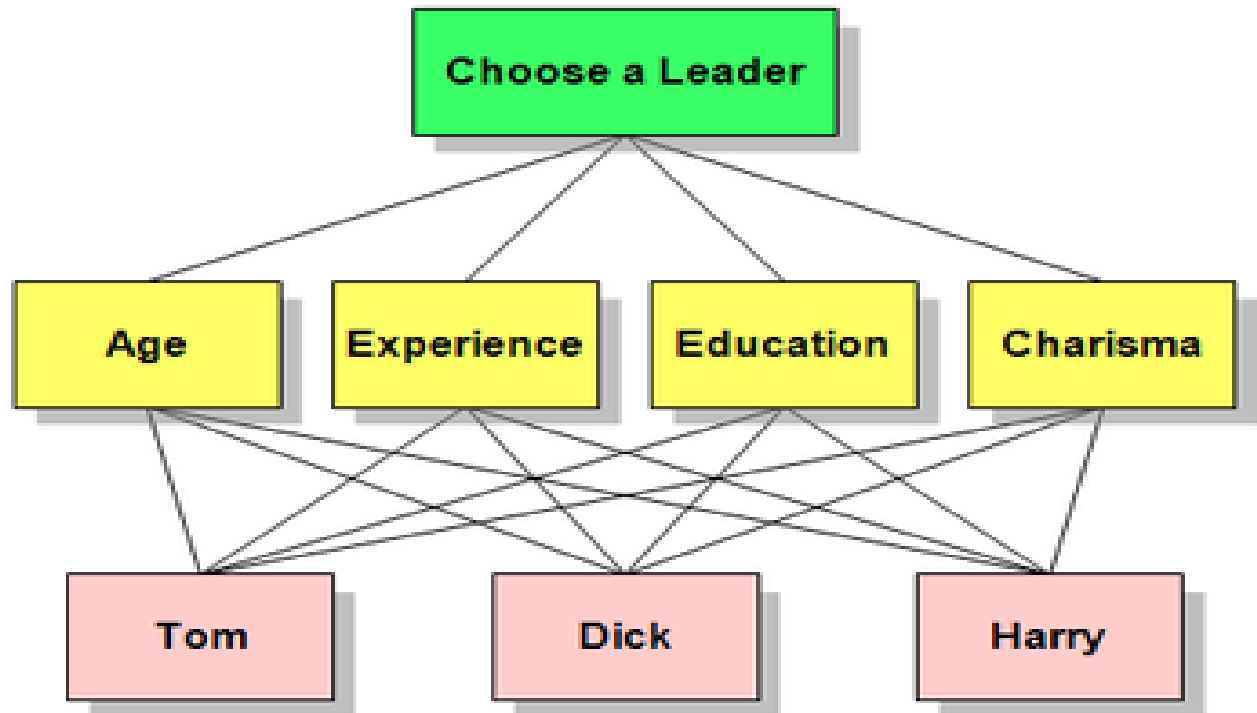
**Charisma**

**Alternatives:**

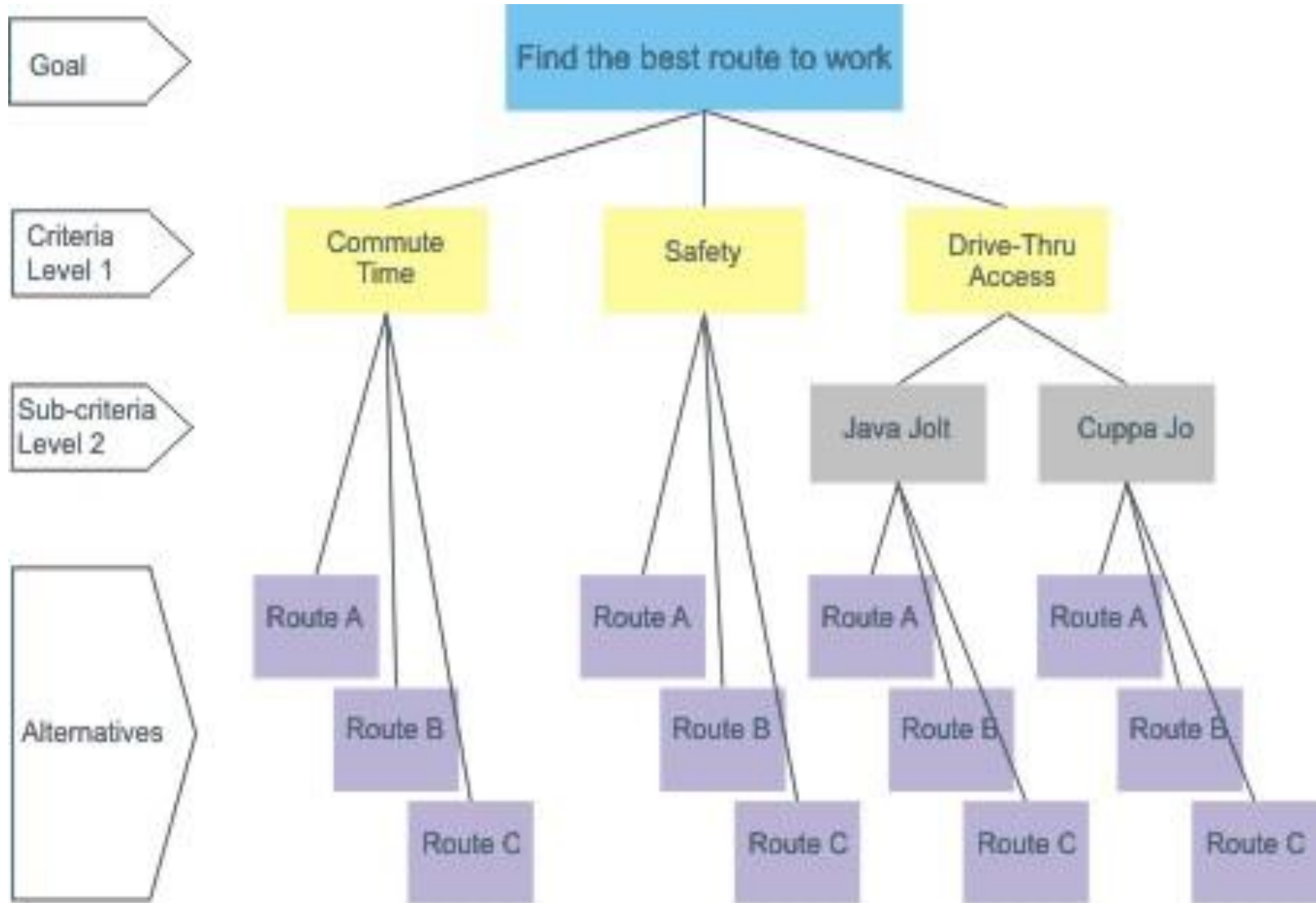
**Tom**

**Dick**

**Harry**



# Best Route Problem

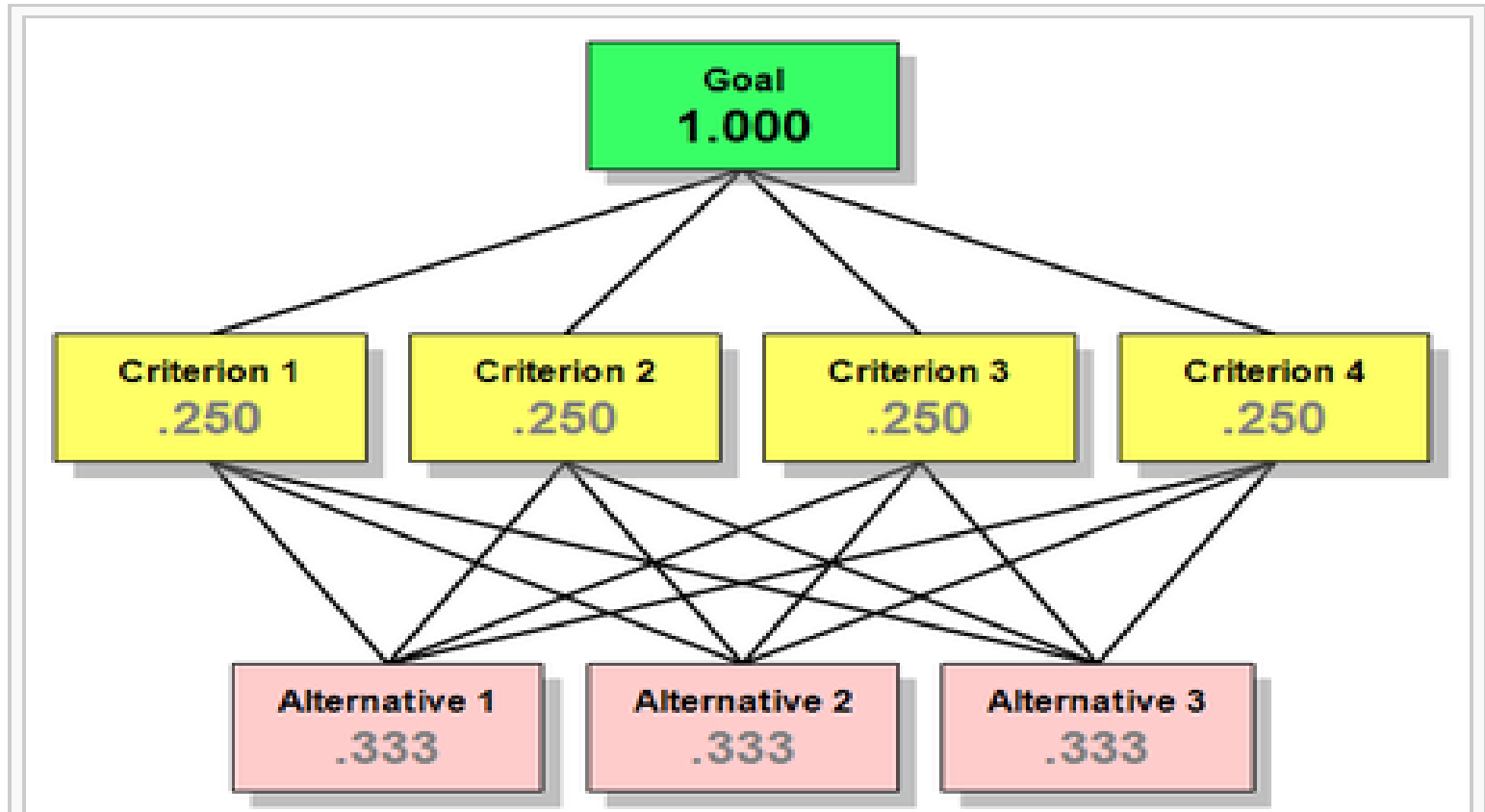


# Priorities

- *Priorities* are numbers associated with the nodes of an AHP hierarchy, they represent the **relative weights** of the nodes in any group
- Like probabilities, priorities are absolute numbers between zero and one; a node with priority .200 has twice the **weight** in reaching the goal as one with priority .100
- Depending on the problem at hand, "weight" can refer to **importance**, or preference, or likelihood, or whatever factor is being considered by the decision makers

# Default Priorities

[equal weight for each criteria and alternative]

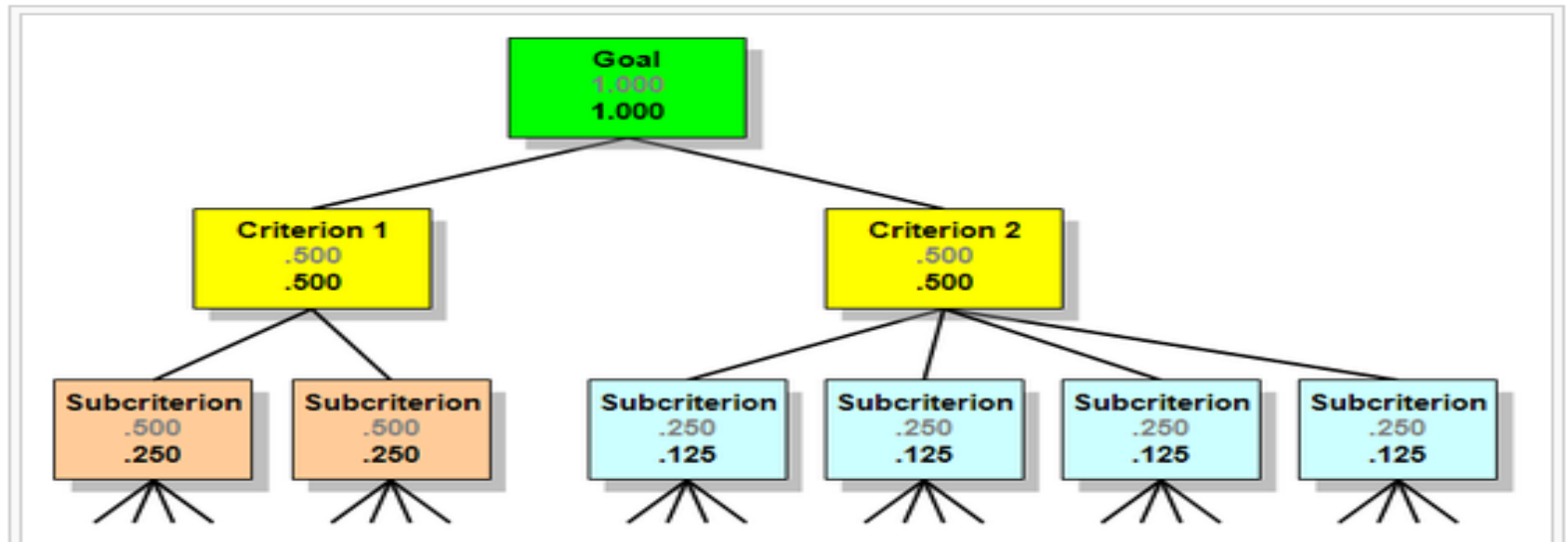


# Priorities (con't)

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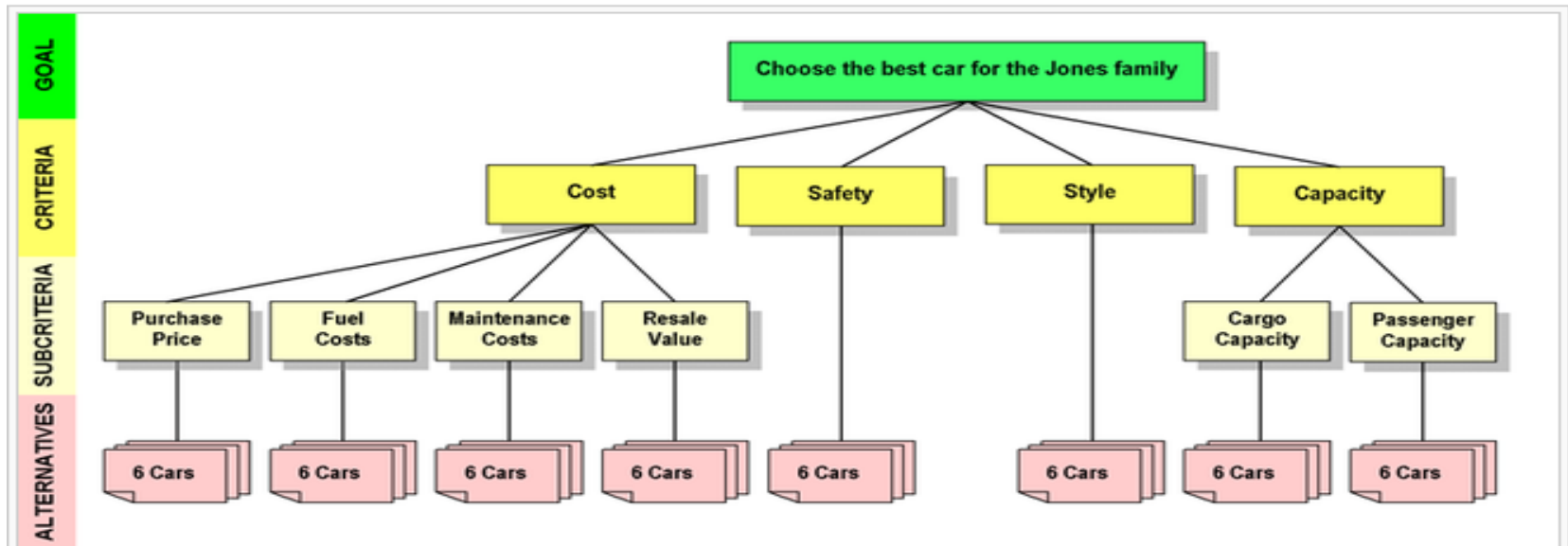
- Priorities are distributed over a hierarchy according to its architecture, and their values depend on the information entered by users of the process
- Priorities of the Goal, the Criteria, and the Alternatives are intimately related, **but need to be considered separately**
- By definition, the priority of the Goal is 1.000 - the priorities of the alternatives always add up to 1.000
- Things can become complicated with multiple levels of Criteria, but if there is only one level, their priorities also add to 1.000

# Sub-Criteria Default Priorities

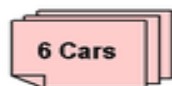


The *local priorities*, shown in gray, represent the relative weights of the nodes within a group of siblings with respect to their parent. You can easily see that the local priorities of each group of Criteria and their sibling Subcriteria add up to 1.000. The *global priorities*, shown in black, are obtained by multiplying the local priorities of the siblings by their parent's global priority. The global priorities for all the subcriteria in the level add up to 1.000. The rule is this: Within a hierarchy, the global priorities of child nodes always add up to the global priority of their parent. Within a group of children, the local priorities add up to 1.000.

# Car Selection Example



**AHP hierarchy for the Jones family car buying decision.** The Goal is green, the Criteria and Subcriteria are yellow, and the Alternatives are pink. All the alternatives (six different models of Hondas) are shown below the lowest level of each criterion. Later in the process, each alternative (each model) will be rated with respect to the criterion or subcriterion directly above it.



=

Accord  
Sedan

Accord  
Hybrid

Pilot  
SUV

CR-V  
SUV

Element  
SUV

Odyssey  
Minivan

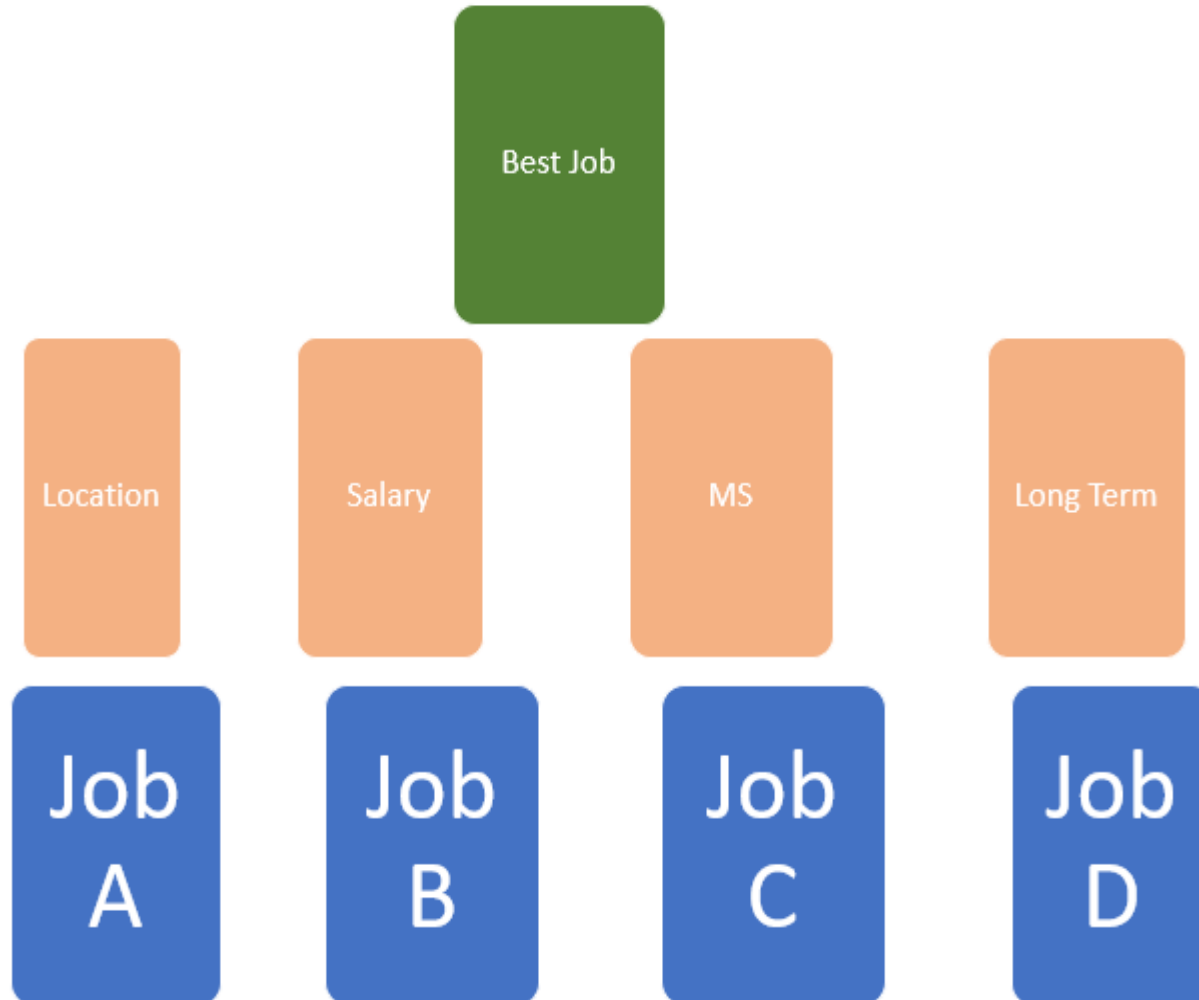
# Simple Example - John Doe's Job Offers

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- John's offers (alternatives) came from Acme Manufacturing, Bankers Bank, Creative Consulting, and Dynamic Decision Making – identified herein as A,B,C,D
- His key decision factors (criteria or objectives) are as location, salary, amount of management science, and long term prospects
- He needs some way to formalize the relative importance, and some way to evaluate each job offer

# John Doe's Job Offers (con't)

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# Importance of Criteria

- The first step in AHP is to ignore the alternatives (jobs) and just decide the relative importance of the criteria
- John does this by comparing each pair of criteria and ranking them on the following scale:
  - Comparing criteria  $i$  and criteria  $j$  (where  $i$  is assumed to be at least as important as  $j$ ), give a value between 1 and 9 as shown on the next side
  - Diagonals are 1 ( $a_{ii} = 1$ )
  - If  $a_{ij} = k$ , then  $a_{ji} = 1/k$

# Pairwise Comparison Values

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- 1 Objectives  $i$  and  $j$  are of equal importance
- 3 Objective  $i$  is weakly more important than  $j$
- 5 Objective  $i$  is strongly more important than  $j$
- 7 Objective  $i$  is very strongly more important than  $j$
- 9 Objective  $i$  is absolutely more important than  $j$
- 2,4,6,8 Intermediate values

# Preferences on Objectives (criteria)

[i.e. salary is 5 time more important that location]

|          | Location | Salary | MS    | Long  |
|----------|----------|--------|-------|-------|
| Location | 1        | $1/5$  | $1/3$ | $1/2$ |
| Salary   | 5        | 1      | 2     | 4     |
| MS       | 3        | $1/2$  | 1     | 3     |
| Long     | 2        | $1/4$  | $1/3$ | 1     |

# Objective Weight

|          | Location | Salary | MS  | Long |
|----------|----------|--------|-----|------|
| Location | 1        | 1/5    | 1/3 | 1/2  |
| Salary   | 5        | 1      | 2   | 4    |
| MS       | 3        | 1/2    | 1   | 3    |
| Long     | 2        | 1/4    | 1/3 | 1    |

- Now, the AHP is going to make some simple calculations to **determine the overall weight that John is assigning to each objective**: this weight will be between 0 and 1, and the total weights will add up to 1
- We do that by taking each entry and **dividing by the sum of the column** it appears in
  - For instance the (Location-Location) entry would end up as:  $1/(1+5+3+2) = 0.091$
- Then we take the **average of each row**

# Weights on Objectives (Criteria)

[sum of averages is 1]

|          | Location | Salary | MS   | Long | Average |
|----------|----------|--------|------|------|---------|
| Location | .091     | .102   | .091 | .059 | .086    |
| Salary   | .455     | .513   | .545 | .471 | .496    |
| MS       | .273     | .256   | .273 | .353 | .289    |
| Long     | .182     | .128   | .091 | .118 | .130    |

|          | Location | Salary | MS   | Long | Average |
|----------|----------|--------|------|------|---------|
| Location | .091     | .102   | .091 | .059 | .086    |
| Salary   | .455     | .513   | .545 | .471 | .496    |
| MS       | .273     | .256   | .273 | .353 | .289    |
| Long     | .182     | .128   | .091 | .118 | .130    |

# Consistency of Decision Making

- This suggests that about half of John's objective weight is on salary, 30% on amount of management science, 13% on long term prospects, and 9% on location
- Now, why does this transformation make sense?
- If we read down the first column in the original matrix, we have the values of each of the objectives, normalized by setting the value of location to be 1; similarly, the second column are the values, normalizing with salary equals 1
- For a perfectly consistent decision maker, each column should be identical, except for the normalization
- By dividing by the total in each column, therefore, **for perfect consistency we would expect identical columns**, with each entry giving the relative weight of the row's objective
- By averaging across each row, we correct for any small inconsistencies in the decision making process

# Using Excel

[sum must be 1]

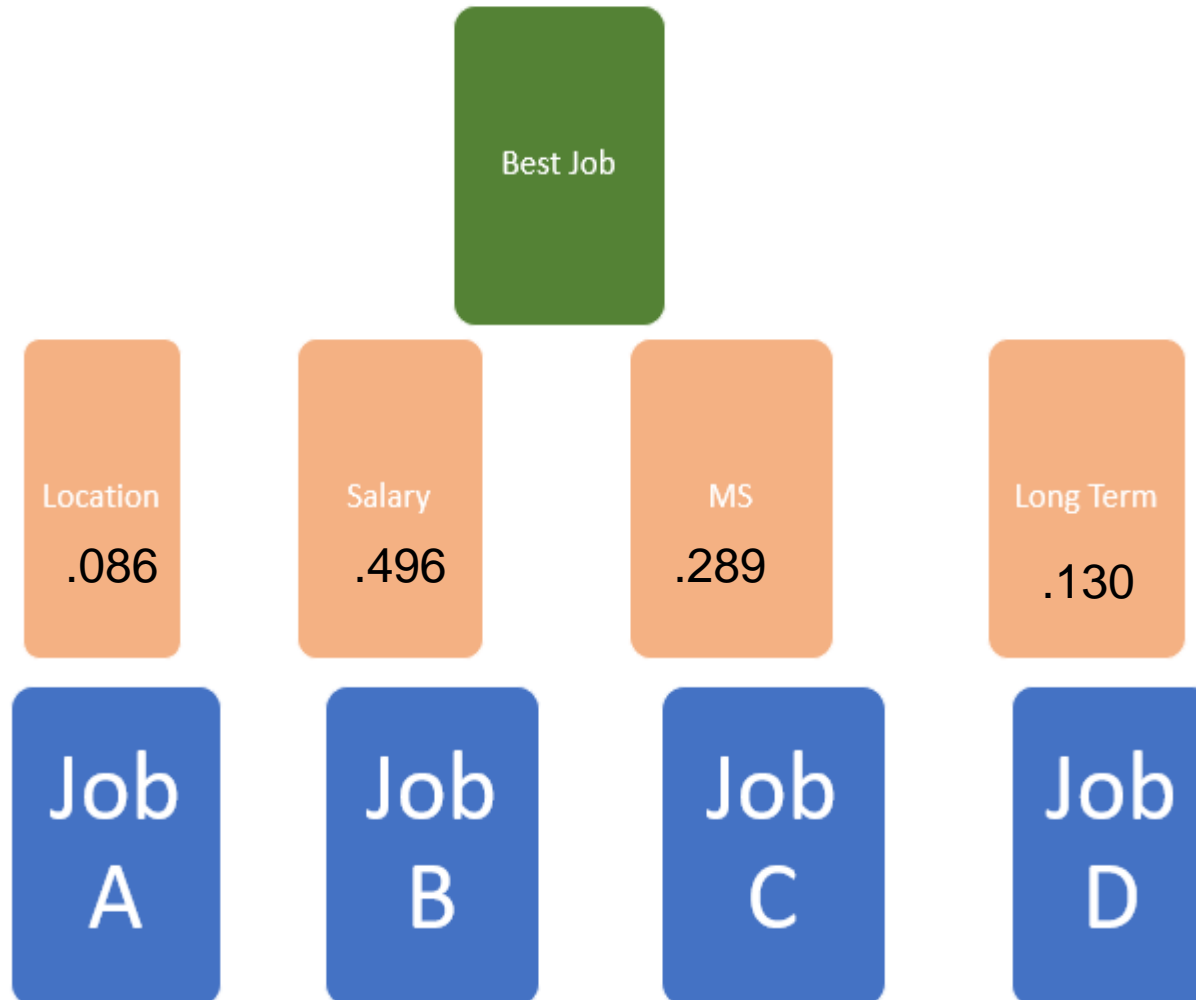
|    | A               | B               | C               | D             | E         | F           | G          |  |
|----|-----------------|-----------------|-----------------|---------------|-----------|-------------|------------|--|
| 1  |                 |                 |                 |               |           |             |            |  |
| 2  |                 |                 |                 |               |           |             |            |  |
| 3  | <b>Criteria</b> |                 |                 |               |           |             |            |  |
| 4  |                 |                 | <b>Location</b> | <b>Salary</b> | <b>MS</b> | <b>Long</b> |            |  |
| 5  |                 | <b>Location</b> | 1               | 0.2           | 0.333333  | 0.5         |            |  |
| 6  |                 | <b>Salary</b>   | 5               | 1             | 2         | 4           |            |  |
| 7  |                 | <b>MS</b>       | 3               | 0.5           | 1         | 3.030303    |            |  |
| 8  |                 | <b>Long</b>     | 2               | 0.25          | 0.33      | 1           |            |  |
| 9  |                 | <b>SUM</b>      | 11              | 1.95          | 3.663333  | 8.530303    |            |  |
| 10 |                 |                 |                 |               |           |             |            |  |
| 11 |                 |                 | <b>Location</b> | <b>Salary</b> | <b>MS</b> | <b>Long</b> | <b>AVE</b> |  |
| 12 |                 | <b>Location</b> | 0.090909        | 0.102564      | 0.090992  | 0.058615    | 0.08577    |  |
| 13 |                 | <b>Salary</b>   | 0.454545        | 0.512821      | 0.545951  | 0.468917    | 0.495558   |  |
| 14 |                 | <b>MS</b>       | 0.272727        | 0.25641       | 0.272975  | 0.35524     | 0.289338   |  |
| 15 |                 | <b>Long</b>     | 0.181818        | 0.128205      | 0.090082  | 0.117229    | 0.129334   |  |
| 16 |                 |                 |                 |               |           | <b>SUM</b>  | 1          |  |
| -- |                 |                 |                 |               |           |             |            |  |

# Using Excel (con't)

|    | A               | B               | C               | D             | E           | F           | G                 |
|----|-----------------|-----------------|-----------------|---------------|-------------|-------------|-------------------|
| 1  |                 |                 |                 |               |             |             |                   |
| 2  |                 |                 |                 |               |             |             |                   |
| 3  | <b>Criteria</b> |                 |                 |               |             |             |                   |
| 4  |                 |                 | <b>Location</b> | <b>Salary</b> | <b>MS</b>   | <b>Long</b> |                   |
| 5  |                 | <b>Location</b> | 1               | =1/C6         | =1/C7       | =1/C8       |                   |
| 6  |                 | <b>Salary</b>   | 5               | 1             | =1/D7       | =1/D8       |                   |
| 7  |                 | <b>MS</b>       | 3               | 0.5           | 1           | =1/E8       |                   |
| 8  |                 | <b>Long</b>     | 2               | 0.25          | 0.33        | 1           |                   |
| 9  |                 | <b>SUM</b>      | =SUM(C5:C8)     | =SUM(D5:D8)   | =SUM(E5:E8) | =SUM(F5:F8) |                   |
| 10 |                 |                 |                 |               |             |             |                   |
| 11 |                 |                 | <b>Location</b> | <b>Salary</b> | <b>MS</b>   | <b>Long</b> | <b>AVE</b>        |
| 12 |                 | <b>Location</b> | =C5/\$C\$9      | =D5/\$D\$9    | =E5/\$E\$9  | =F5/\$F\$9  | =AVERAGE(C12:F12) |
| 13 |                 | <b>Salary</b>   | =C6/\$C\$9      | =D6/\$D\$9    | =E6/\$E\$9  | =F6/\$F\$9  | =AVERAGE(C13:F13) |
| 14 |                 | <b>MS</b>       | =C7/\$C\$9      | =D7/\$D\$9    | =E7/\$E\$9  | =F7/\$F\$9  | =AVERAGE(C14:F14) |
| 15 |                 | <b>Long</b>     | =C8/\$C\$9      | =D8/\$D\$9    | =E8/\$E\$9  | =F8/\$F\$9  | =AVERAGE(C15:F15) |
| 16 |                 |                 |                 |               |             | <b>SUM</b>  | =SUM(G12:G15)     |

# John Doe's Job Offers (con't)

|          | Location | Salary | MS   | Long | Average |
|----------|----------|--------|------|------|---------|
| Location | .091     | .102   | .091 | .059 | .086    |
| Salary   | .455     | .513   | .545 | .471 | .496    |
| MS       | .273     | .256   | .273 | .353 | .289    |
| Long     | .182     | .128   | .091 | .118 | .130    |



# Exercise

|          | Location      | Salary      | MS          | Long        |                   |
|----------|---------------|-------------|-------------|-------------|-------------------|
| Location | 1             | =1/C6       | =1/C7       | =1/C8       |                   |
| Salary   | 5             | 1           | =1/D7       | =1/D8       |                   |
| MS       | 3             | 0.5         | 1           | =1/E8       |                   |
| Long     | 2             | 0.25        | 0.33        | 1           |                   |
| SUM      | =SUM(C5:C8)   | =SUM(D5:D8) | =SUM(E5:E8) | =SUM(F5:F8) |                   |
|          | Location      | Salary      | MS          | Long        | AVE               |
| Location | =C5/\$C\$9    | =D5/\$D\$9  | =E5/\$E\$9  | =F5/\$F\$9  | =AVERAGE(C12:F12) |
| Salary   | =C6/\$C\$9    | =D6/\$D\$9  | =E6/\$E\$9  | =F6/\$F\$9  | =AVERAGE(C13:F13) |
| MS       | =C7/\$C\$9    | =D7/\$D\$9  | =E7/\$E\$9  | =F7/\$F\$9  | =AVERAGE(C14:F14) |
| Long     | =C8/\$C\$9    | =D8/\$D\$9  | =E8/\$E\$9  | =F8/\$F\$9  | =AVERAGE(C15:F15) |
| SUM      | =SUM(G12:G15) |             |             |             |                   |

- In Excel, perform a pair-wise criteria evaluation for the relative importance of how you spend your awake time in:
  - Work
  - Learning (academic, advancement, etc.)
  - Recreation (sports, hobbies, games, TV, etc.)
  - Relationships (significant other, family, friends)
  - Devotion, Service, Reflection



# Exercise

|          | Location      | Salary      | MS          | Long        |                   |
|----------|---------------|-------------|-------------|-------------|-------------------|
| Location | 1             | =1/C6       | =1/C7       | =1/C8       |                   |
| Salary   | 5             | 1           | =1/D7       | =1/D8       |                   |
| MS       | 3             | 0.5         | 1           | =1/E8       |                   |
| Long     | 2             | 0.25        | 0.33        | 1           |                   |
| SUM      | =SUM(C5:C8)   | =SUM(D5:D8) | =SUM(E5:E8) | =SUM(F5:F8) |                   |
|          | Location      | Salary      | MS          | Long        | AVE               |
| Location | =C5/\$C\$9    | =D5/\$D\$9  | =E5/\$E\$9  | =F5/\$F\$9  | =AVERAGE(C12:F12) |
| Salary   | =C6/\$C\$9    | =D6/\$D\$9  | =E6/\$E\$9  | =F6/\$F\$9  | =AVERAGE(C13:F13) |
| MS       | =C7/\$C\$9    | =D7/\$D\$9  | =E7/\$E\$9  | =F7/\$F\$9  | =AVERAGE(C14:F14) |
| Long     | =C8/\$C\$9    | =D8/\$D\$9  | =E8/\$E\$9  | =F8/\$F\$9  | =AVERAGE(C15:F15) |
| SUM      | =SUM(G12:G15) |             |             |             |                   |

- Now tabulate how many weekly hours you actually spend on each; total should be about 896 (8 x 16 x 7):
  - Work
  - Learning (academic, advancement, etc.)
  - Recreation (sports, hobbies, games, TV, etc.)
  - Relationships (significant other, family, friends)
  - Devotion, Service, Reflection
- Compare the ratios with your priorities



# Evaluate Alternatives

- The next step is to evaluate all the alternatives (jobs – A,B,C,D) on each objective
- For instance, if we take Location, and if we prefer to be in the northeast (and preferably Boston), and say the jobs are located in Pittsburgh, New York, Boston, and San Francisco respectively, then we might get the following matrix:



# Location Scores

[i.e. B is twice as good as A **in regard to location**]

|   | A   | B   | C   | D |
|---|-----|-----|-----|---|
| A | 1   | 1/2 | 1/3 | 5 |
| B | 2   | 1   | 1/2 | 7 |
| C | 3   | 2   | 1   | 9 |
| D | 1/5 | 1/7 | 1/9 | 1 |

# Relative (normalized) Location Scores

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|   | A    | B    | C    | D    | Average |
|---|------|------|------|------|---------|
| A | .161 | .137 | .171 | .227 | .174    |
| B | .322 | .275 | .257 | .312 | .293    |
| C | .484 | .549 | .514 | .409 | .489    |
| D | .032 | .040 | .057 | .045 | .044    |

# In Excel:

|    |          |     |           |           |           |            |           |  |
|----|----------|-----|-----------|-----------|-----------|------------|-----------|--|
| 18 | Location |     |           |           |           |            |           |  |
| 19 |          |     | A         | B         | C         | D          |           |  |
| 20 |          | A   | 1.0000000 | 0.5000000 | 0.3333333 | 5.0000000  |           |  |
| 21 |          | B   | 2.0000000 | 1.0000000 | 0.5000000 | 7.0000000  |           |  |
| 22 |          | C   | 3.0000000 | 2.0000000 | 1.0000000 | 9.0000000  |           |  |
| 23 |          | D   | 0.2000000 | 0.1428571 | 0.1111111 | 1.0000000  |           |  |
| 24 |          | SUM | 6.2000000 | 3.6428571 | 1.9444444 | 22.0000000 |           |  |
| 25 |          |     |           |           |           |            |           |  |
| 26 |          |     | A         | B         | C         | D          | AVE       |  |
| 27 |          | A   | 0.1612903 | 0.1372549 | 0.1714286 | 0.2272727  | 0.1743116 |  |
| 28 |          | B   | 0.3225806 | 0.2745098 | 0.2571429 | 0.3181818  | 0.2931038 |  |
| 29 |          | C   | 0.4838710 | 0.5490196 | 0.5142857 | 0.4090909  | 0.4890668 |  |
| 30 |          | D   | 0.0322581 | 0.0392157 | 0.0571429 | 0.0454545  | 0.0435178 |  |

# Relative Location Scores (con't)

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- In words, of the the total “Location Value” available, Job C has about 50%, B has about 30%, A has about 17% and D has about 4%
- We now go through a similar process with Salary, amount of MS, and long term prospects
- Suppose the relative values for those objectives can be given as follows:

# Relative Scores (average column) for Each Objective

|          | A    | B    | C    | D    |
|----------|------|------|------|------|
| Location | .174 | .293 | .489 | .044 |
| Salary   | .050 | .444 | .312 | .194 |
| MS       | .210 | .038 | .354 | .398 |
| Long     | .510 | .012 | .290 | .188 |



|   | A    | B    | C    | D    | Average |
|---|------|------|------|------|---------|
| A | .161 | .137 | .171 | .227 | .174    |
| B | .322 | .275 | .257 | .312 | .293    |
| C | .484 | .549 | .514 | .409 | .489    |
| D | .032 | .040 | .057 | .045 | .044    |

Location Table

# Results...

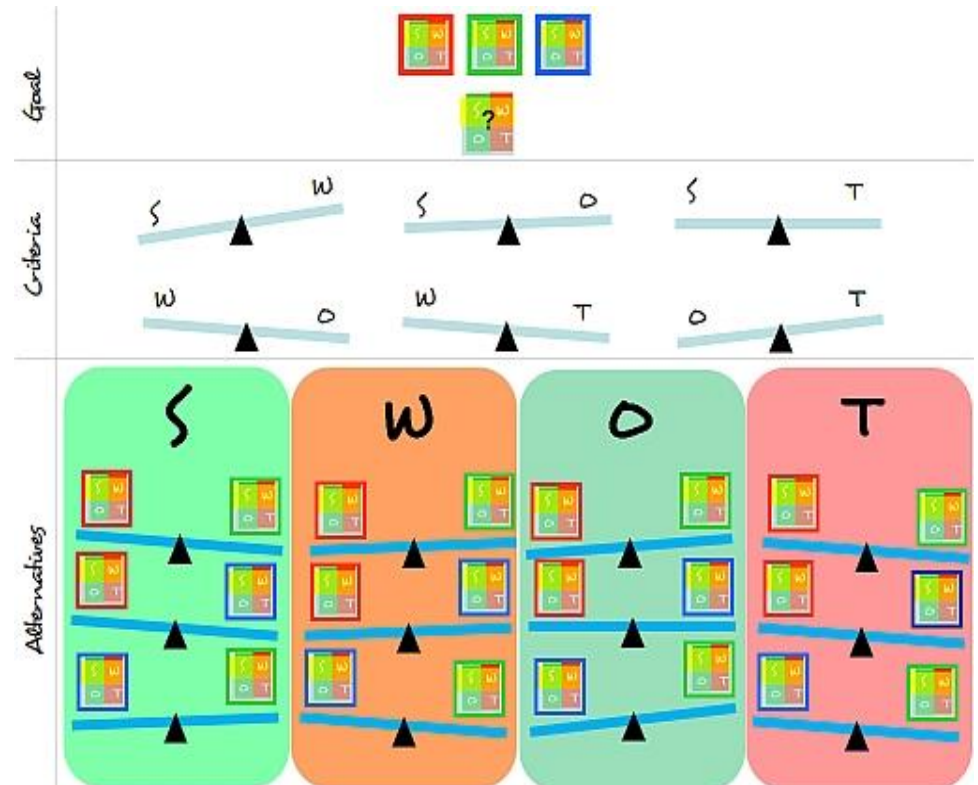
|          | Job Table |      |      |      |
|----------|-----------|------|------|------|
|          | A         | B    | C    | D    |
| Location | .174      | .293 | .489 | .044 |
| Salary   | .050      | .444 | .312 | .194 |
| MS       | .210      | .038 | .354 | .398 |
| Long     | .510      | .012 | .290 | .188 |

|          | Criteria Table |        |      |      |         |
|----------|----------------|--------|------|------|---------|
|          | Location       | Salary | MS   | Long | Average |
| Location | .091           | .102   | .091 | .059 | .086    |
| Salary   | .455           | .513   | .545 | .471 | .496    |
| MS       | .273           | .256   | .273 | .353 | .289    |
| Long     | .182           | .128   | .091 | .118 | .130    |

- Recalling our overall weights for the objectives, we can now get a value for each job
  - The value for Acme Manufacturing (A) is:
    - $.174 \times .086 + .05 \times .496 + .21 \times .289 + .51 \times .13 = .164$
  - Similarly, the value Bankers Bank (B) is  $.256$
  - The value for Creative Consultants (C) is  $.335$
  - The value for Dynamic Decision (D) is  $.238$
  - Creative Consultants it is the choice !

# AHP Summary

- The Analytic Hierarchy Process is a method for formalizing decision making where there are a limited number of choices but each has a number of criteria and it is difficult to quantify some of those criteria



# AHP Summary (con't)

- Note in this example, we did not collect any data (like miles from a preferred point or salary numbers)
- Instead, we use phrases like "much more important than" to extract the decision makers preferences

Table 3. Computation of Hypothetical Sample Difference - AHP

| Analytic Hierarchy Process          |              |                  |                 |            |              |
|-------------------------------------|--------------|------------------|-----------------|------------|--------------|
| Subject                             | ALTERNATIVES | Stage 1 - (BASE) | Stage 2 - (AHP) | Difference | Diff Squared |
| 1                                   | Peotone      | .35              | .15             | .20        | .0400        |
|                                     | Bi-State     | .40              | .40             | .00        | .0000        |
|                                     | Gary         | .25              | .45             | -.20       | .0400        |
| 2                                   | Peotone      | .10              | .40             | -.30       | .0900        |
|                                     | Bi-State     | .25              | .35             | -.10       | .0100        |
|                                     | Gary         | .65              | .25             | .40        | .1600        |
| 3                                   | Peotone      | .10              | .25             | -.15       | .0225        |
|                                     | Bi-State     | .10              | .35             | -.25       | .0625        |
|                                     | Gary         | .80              | .40             | .40        | .1600        |
| Mean of Differences Squared → .1950 |              |                  |                 |            |              |

# AHP Summary (con't)

- Despite the rather arbitrary aspects of the procedure, however, it does provide useful insight into the tradeoffs embedded in a complex decision making problem
- It also has the advantage of getting “buy-in” from the participants in determining the weights



# AHP in Evaluating Business Initiatives or Projects



[VIEW LARGER](#) 20

# Software for AHP: Expert Choice

## ■ No Excel-QM nor QM module for AHP

The screenshot displays the Expert Choice website interface. At the top, the navigation bar includes links for 'About Us', 'News & Events', 'Contact', 'Support', 'Login', 'Comparison Suite', 'Academic Program', and 'Partner Program'. A prominent banner reads 'Move Forward Now 1.800.447.0506' with a 'Go' button. Below this, a secondary navigation menu features 'Home', 'How Our Software Helps', 'Products & Services', 'Markets We Serve', 'Clients & Successes', and 'Resource Center'. The 'How Our Software Helps' section is expanded, listing categories such as 'Transparency in the Federal Government', 'Project & Product Management', 'Strategic Planning & Budgeting', 'Vendor & Human Resource Management', 'Decision Support & Process Management', and 'Market Research & Insights'. The main content area is titled 'Collaborative Decision-Making for a Complex World' and includes a paragraph about the challenges of complex decision-making. A graph titled 'Problems our Software Addresses' plots 'Importance' against 'Number of Participants', showing four quadrants: 'Reason', 'Agreeing', 'Intuition', and 'Polling'. The sidebar on the right features a testimonial from Randy Howard, a 'See More' link, a 'Download Case Study' link, and a list of product categories with expandable icons. The Windows taskbar at the bottom shows the 'start' button and several open applications, including 'Eudora - [In]', 'Microsoft Of...', 'Overcoming Yo...', 'Microsoft Power...', and 'PowerPoint Slid...', with the system clock indicating 11:58 AM.

expertchoice®

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Move Forward Now 1.800.447.0506  Go

Home How Our Software Helps Products & Services Markets We Serve Clients & Successes Resource Center

» Transparency in the Federal Government

» Project & Product Management

» Strategic Planning & Budgeting

» Vendor & Human Resource Management

» Decision Support & Process Management

» Market Research & Insights

See how our products work in your world

Select a Market

### Collaborative Decision-Making for a Complex World

More variables. More information. More stakeholders. More of everything—except time.

That's why successful organizations in government and industry depend on Expert Choice for rapid convergence of experience, intuition, and data to collaborate and make decisions with alignment, buy-in, and confidence.

IMPORTANCE

Reason Agreeing

Intuition Polling

NUMBER OF PARTICIPANTS

Problems our Software Addresses

Conflicting Priorities

#### How Our Software Helps

Project & Product Management

"We were able to go from about 120 projects down to roughly 30 projects, and the decisions were more strategically focused. We've taken a portfolio that would have cost \$100 million and saved nearly \$50 million."

**Randy Howard**  
Technical Project Director, Project Portfolio Management, AOL

» See More | » Download Case Study

Strategic Planning & Budgeting

Vendor & Human Resource Management

Decision Support & Process Management

Market Research & Insights

start

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11:58 AM

# Homework

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- Textbook online module 1
- **Project 13 (in Excel)**
  - Use AHP for your new car selection
  - Criteria (one level):
    - Price
    - Gas Mileage (MPG)
    - Safety
    - Capacity
  - Alternatives – pick 3 autos to evaluate