Management Science

Analytic Hierarchy Process

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Analytic Hierarchy Process (AHP)

- This is a course about quantitative methods
- However, it is often difficult to conceptualize or quantify all the different elements of a problem
- The AHP was formulated to counter those situations, and is a mathematically-based theory
- It employs two key aspects:
  - (1) data from the various variables that make up the decision
  - (2) judgments about those variables

- How do you determine the best route to school/work each day ?

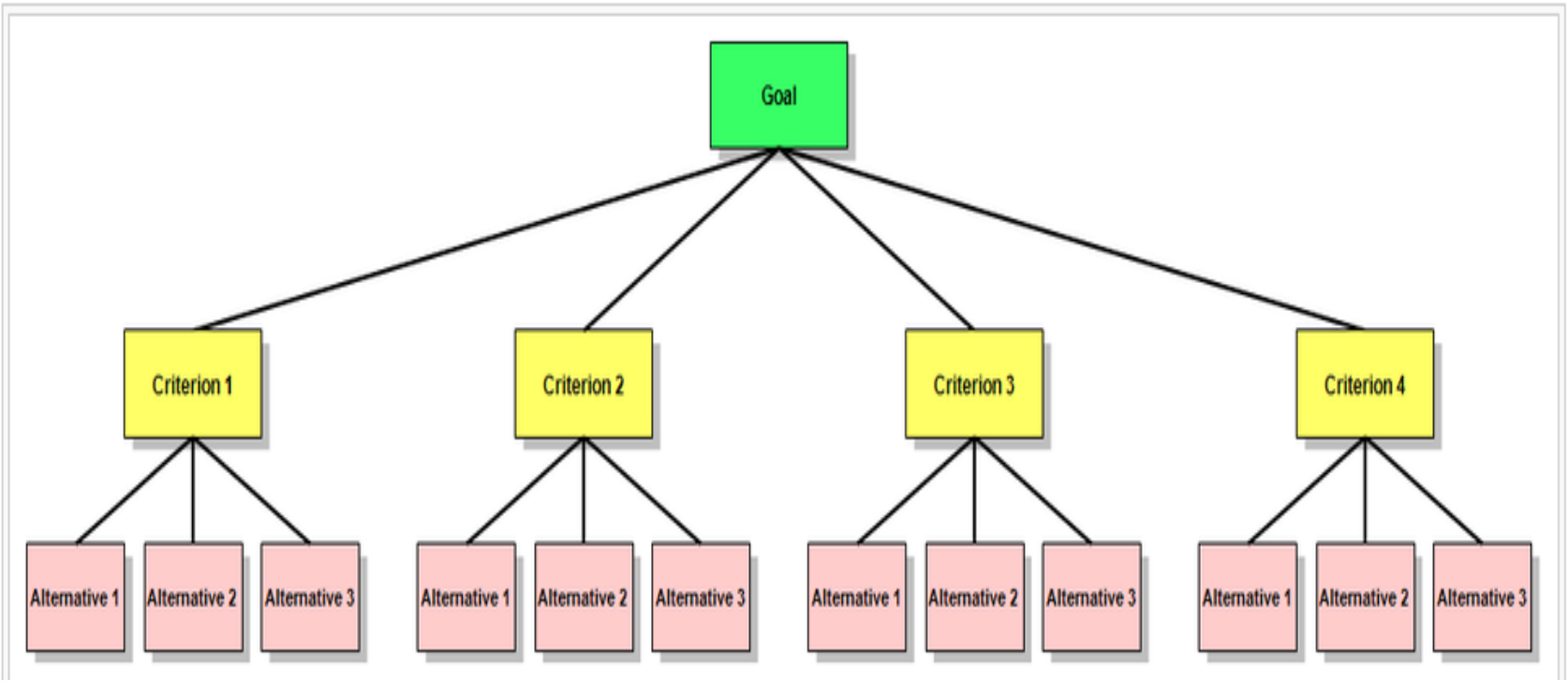
Do not look ahead !

- The fastest route ?
- The cheapest route ?
- The safest route ?
- The most scenic route ?
- The route with the most coffee drive thru's ?

Analytic Hierarchy Process (con't)

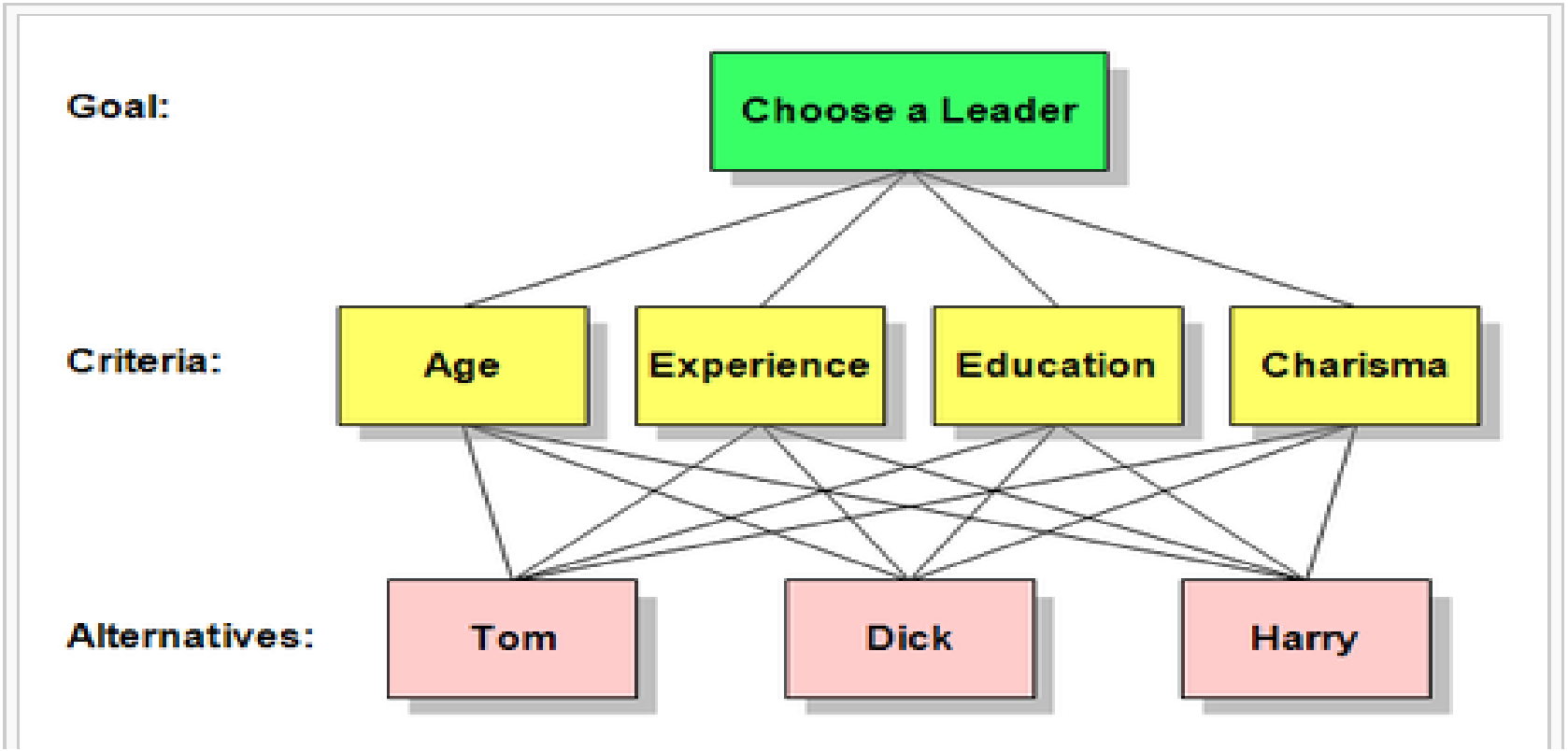
- The AHP requires taking the following steps:
  1. Structuring the decision into a hierarchical model
  2. Pairwise comparison of all objects and alternative solutions
- The form of the model has four elements:
  1. *Goal* – the desired outcome
  2. *Criteria* – elements that comprise the goal (objectives)
  3. *Subcriteria* – elements inside the criteria
  4. *Alternatives* – solutions or choices available
- This format allows decision makers to examine every part of a complex problem

Analytic Hierarchy Process (con't)

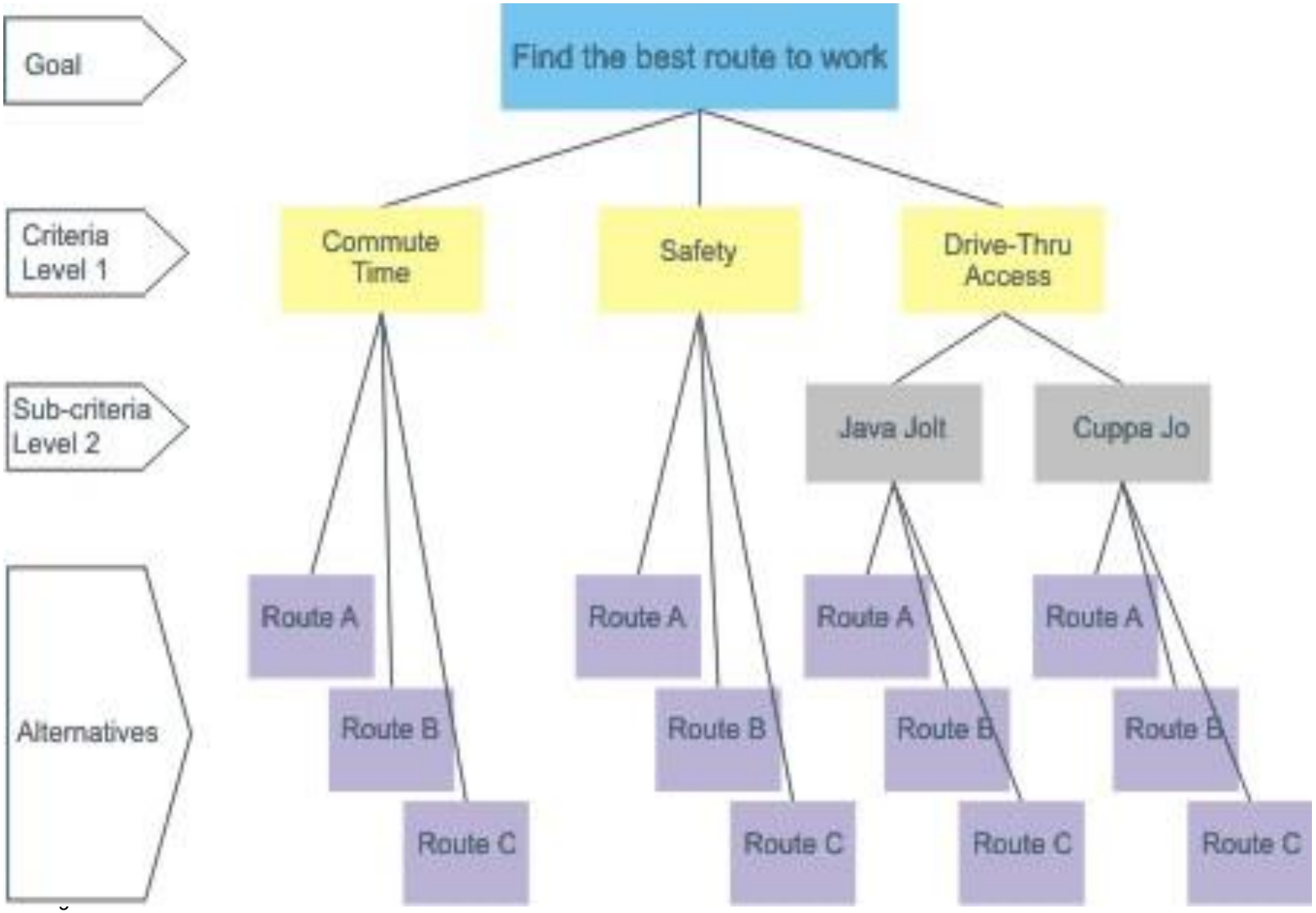


AHP will evaluate each alternative for each criteria.

AHP Example



Best Route Problem

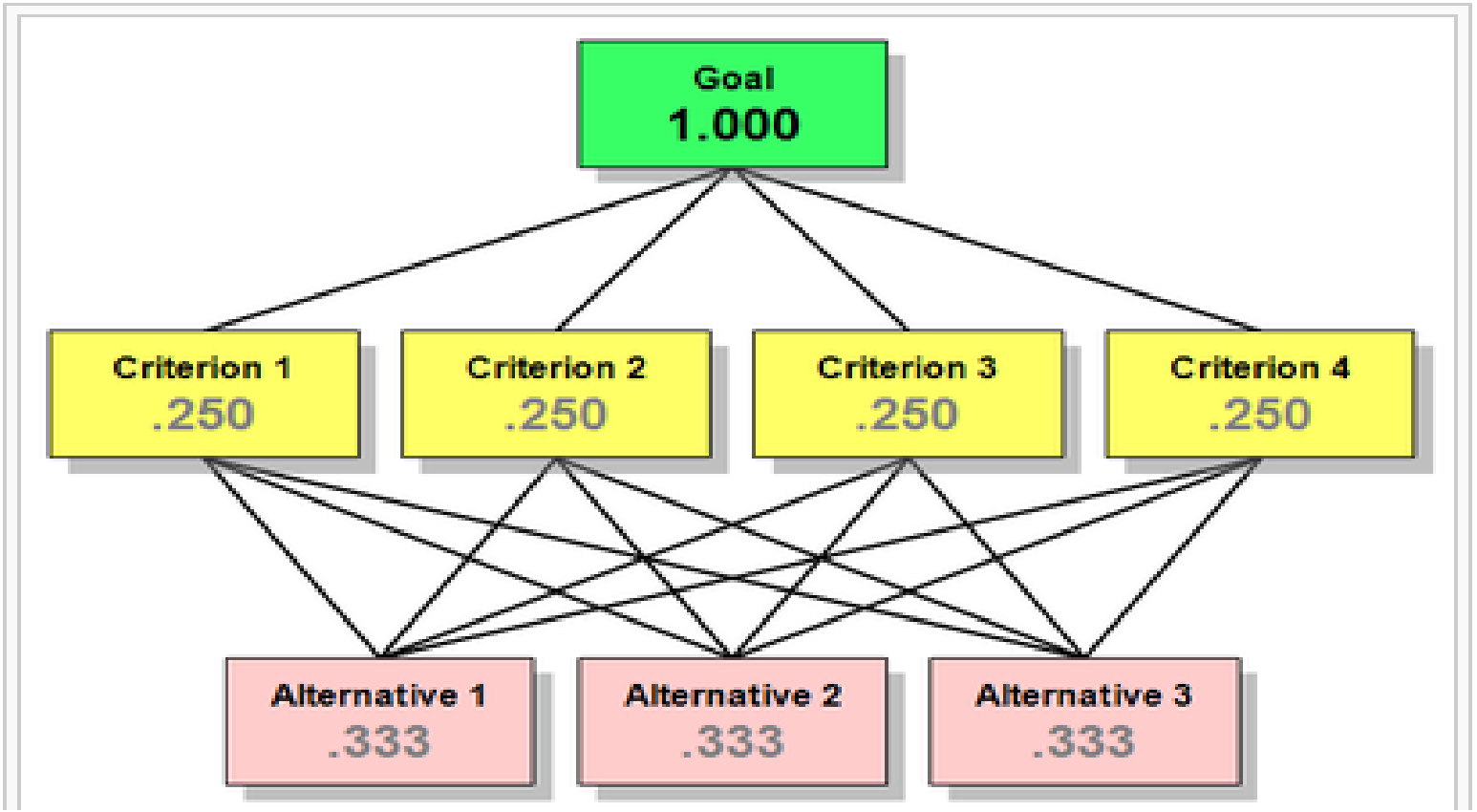


Priorities

- *Priorities* are numbers associated with the nodes of an AHP hierarchy, they represent the relative weights of the nodes in any group
- Like probabilities, priorities are absolute numbers between zero and one; a node with priority .200 has twice the weight in reaching the goal as one with priority .100
- Depending on the problem at hand, "weight" can refer to importance, or preference, or likelihood, or whatever factor is being considered by the decision makers

Default Priorities

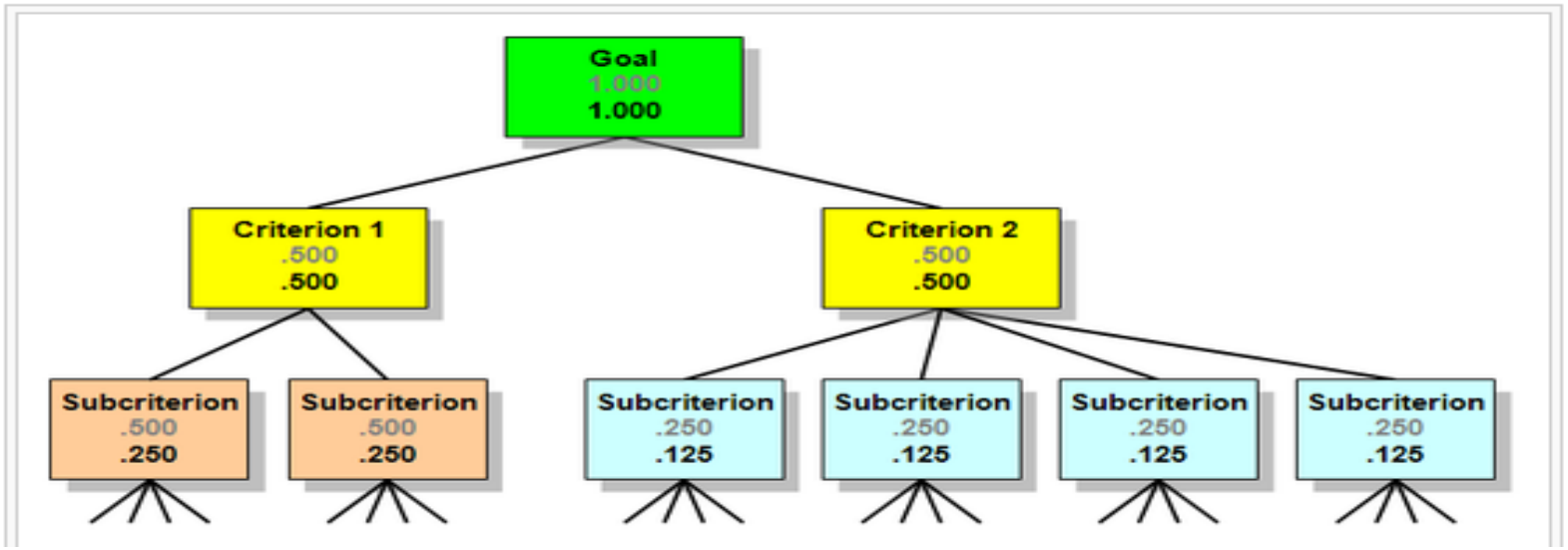
[equal weight for each criteria and alternative]



Priorities (con't)

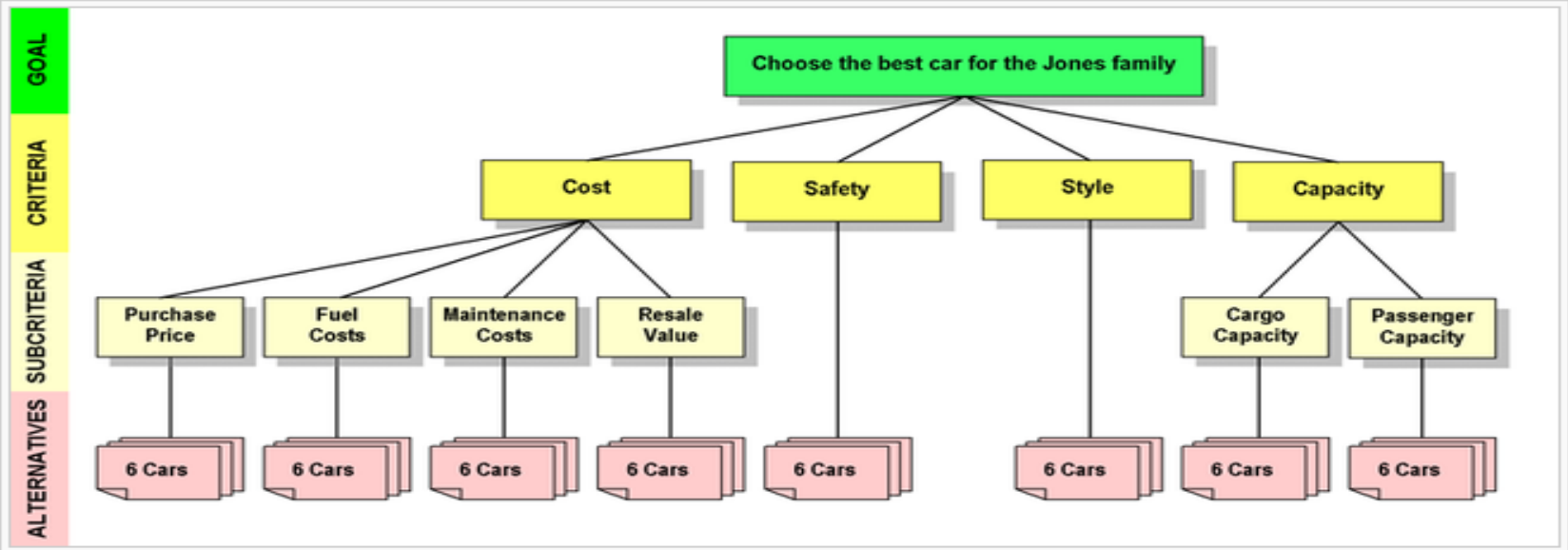
- Priorities are distributed over a hierarchy according to its architecture, and their values depend on the information entered by users of the process
- Priorities of the Goal, the Criteria, and the Alternatives are intimately related, but need to be considered separately
- By definition, the priority of the Goal is 1.000 - the priorities of the alternatives always add up to 1.000
- Things can become complicated with multiple levels of Criteria, but if there is only one level, their priorities also add to 1.000

Sub-Criteria Default Priorities

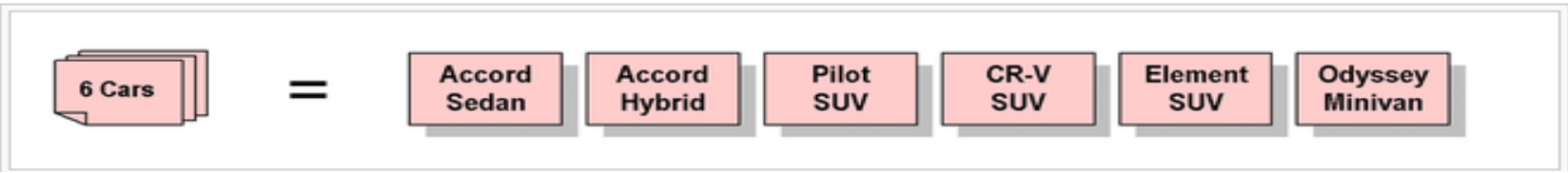


The *local priorities*, shown in gray, represent the relative weights of the nodes within a group of siblings with respect to their parent. You can easily see that the local priorities of each group of Criteria and their sibling Subcriteria add up to 1.000. The *global priorities*, shown in black, are obtained by multiplying the local priorities of the siblings by their parent's global priority. The global priorities for all the subcriteria in the level add up to 1.000. The rule is this: Within a hierarchy, the global priorities of child nodes always add up to the global priority of their parent. Within a group of children, the local priorities add up to 1.000.

Car Selection Example



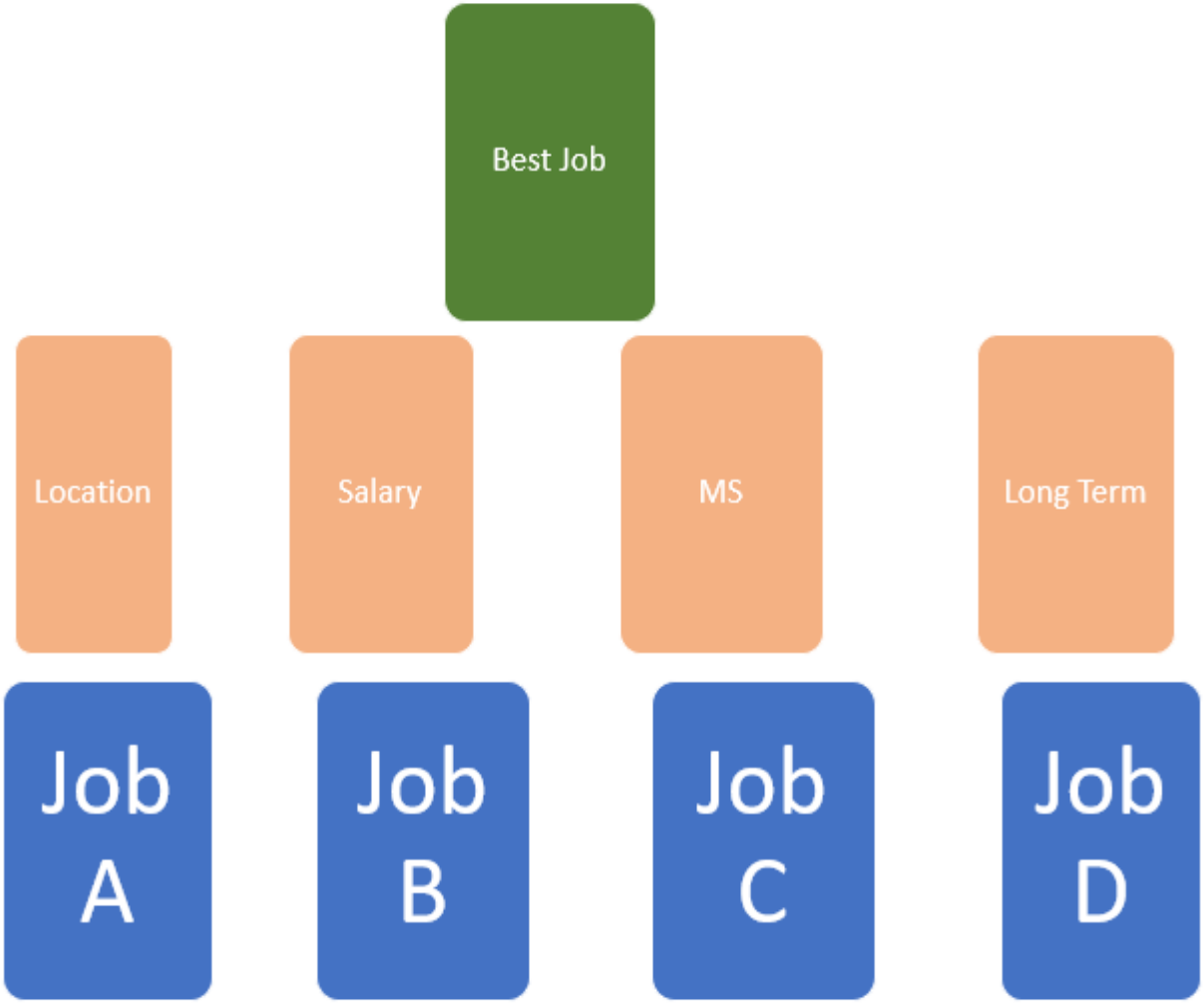
AHP hierarchy for the Jones family car buying decision. The Goal is green, the Criteria and Subcriteria are yellow, and the Alternatives are pink. All the alternatives (six different models of Hondas) are shown below the lowest level of each criterion. Later in the process, each alternative (each model) will be rated with respect to the criterion or subcriterion directly above it.



Simple Example - John Doe's Job Offers

- John's offers (alternatives) came from Acme Manufacturing, Bankers Bank, Creative Consulting, and Dynamic Decision Making – identified herein as A,B,C,D
- His key decision factors (criteria or objectives) are as location, salary, amount of management science, and long term prospects
- He needs some way to formalize the relative importance, and some way to evaluate each job offer

John Doe's Job Offers (con't)



Importance of Criteria

- The first step in AHP is to ignore the alternatives (jobs) and just decide the relative importance of the criteria
- John does this by comparing each pair of criteria and ranking them on the following scale:
  - Comparing criteria *i* and criteria *j* (where *i* is assumed to be at least as important as *j*), give a value between 1 and 9 as shown on the next side
  - Diagonals are 1 ($a_{ii} = 1$)
  - If $a_{ij} = k$, then $a_{ji} = 1/k$

Pairwise Comparison Values

1	Objectives i and j are of equal importance
3	Objective i is weakly more important than j
5	Objective i is strongly more important than j
7	Objective i is very strongly more important than j
9	Objective i is absolutely more important than j
2,4,6,8	Intermediate values

Preferences on Objectives (criteria)

[i.e. salary is 5 time more important that location]

	Location	Salary	MS	Long
Location	1	1/5	1/3	1/2
Salary	5	1	2	4
MS	3	1/2	1	3
Long	2	1/4	1/3	1

Objective Weight

	Location	Salary	MS	Long
Location	1	1/5	1/3	1/2
Salary	5	1	2	4
MS	3	1/2	1	3
Long	2	1/4	1/3	1

- Now, the AHP is going to make some simple calculations to determine the overall weight that John is assigning to each objective: this weight will be between 0 and 1, and the total weights will add up to 1
- We do that by taking each entry and dividing by the sum of the column it appears in
  - For instance the (Location-Location) entry would end up as: 1/(1+5+3+2) = 0.091
- Then we take the average of each row

Weights on Objectives (Criteria)

[sum of averages is 1]

	Location	Salary	MS	Long	Average
Location	.091	.102	.091	.059	.086
Salary	.455	.513	.545	.471	.496
MS	.273	.256	.273	.353	.289
Long	.182	.128	.091	.118	.130

Consistency of Decision Making

	Location	Salary	MS	Long	Average
Location	.091	.102	.091	.059	.086
Salary	.455	.513	.545	.471	.496
MS	.273	.256	.273	.353	.289
Long	.182	.128	.091	.118	.130

- This suggests that about half of John's objective weight is on salary, 30% on amount of management science, 13% on long term prospects, and 9% on location
- Now, why does this transformation make sense?
- If we read down the first column in the original matrix, we have the values of each of the objectives, normalized by setting the value of location to be 1; similarly, the second column are the values, normalizing with salary equals 1
- For a perfectly consistent decision maker, each column should be identical, except for the normalization
- By dividing by the total in each column, therefore, for perfect consistency we would expect identical columns, with each entry giving the relative weight of the row's objective
- By averaging across each row, we correct for any small inconsistencies in the decision making process

Using Excel

[sum must be 1]

	A	B	C	D	E	F	G
1							
2							
3	**Criteria**						
4			**Location**	**Salary**	**MS**	**Long**	
5		**Location**	1	0.2	0.333333	0.5	
6		**Salary**	5	1	2	4	
7		**MS**	3	0.5	1	3.030303	
8		**Long**	2	0.25	0.33	1	
9		**SUM**	11	1.95	3.663333	8.530303	
10							
11			**Location**	**Salary**	**MS**	**Long**	**AVE**
12		**Location**	0.090909	0.102564	0.090992	0.058615	0.08577
13		**Salary**	0.454545	0.512821	0.545951	0.468917	0.495558
14		**MS**	0.272727	0.25641	0.272975	0.35524	0.289338
15		**Long**	0.181818	0.128205	0.090082	0.117229	0.129334
16						**SUM**	1

Using Excel (con't)

	A	B	C	D	E	F	G
1							
2							
3	**Criteria**						
4			**Location**	**Salary**	**MS**	**Long**	
5		**Location**	1	=1/C6	=1/C7	=1/C8	
6		**Salary**	5	1	=1/D7	=1/D8	
7		**MS**	3	0.5	1	=1/E8	
8		**Long**	2	0.25	0.33	1	
9		**SUM**	=SUM(C5:C8)	=SUM(D5:D8)	=SUM(E5:E8)	=SUM(F5:F8)	
10							
11			**Location**	**Salary**	**MS**	**Long**	**AVE**
12		**Location**	=C5/$C$9	=D5/$D$9	=E5/$E$9	=F5/$F$9	=AVERAGE(C12:F12)
13		**Salary**	=C6/$C$9	=D6/$D$9	=E6/$E$9	=F6/$F$9	=AVERAGE(C13:F13)
14		**MS**	=C7/$C$9	=D7/$D$9	=E7/$E$9	=F7/$F$9	=AVERAGE(C14:F14)
15		**Long**	=C8/$C$9	=D8/$D$9	=E8/$E$9	=F8/$F$9	=AVERAGE(C15:F15)
16						**SUM**	=SUM(G12:G15)

John Doe's Job Offers (con't)

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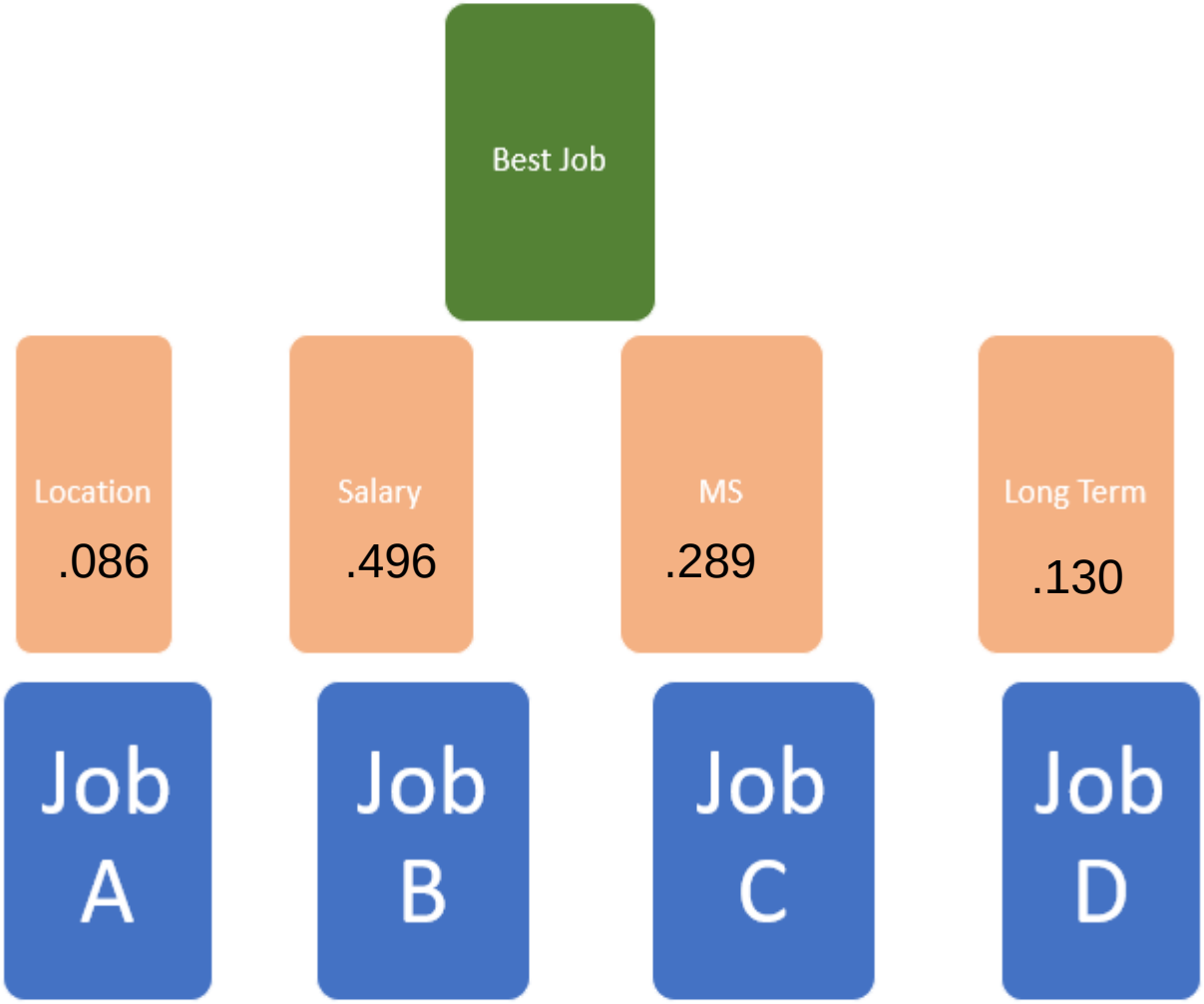

Exercise

	Location	Salary	MS	Long	
Location	1	=1/C6	=1/C7	=1/C8	
Salary	5	1	=1/D7	=1/D8	
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SUM	=SUM(C5:C8)	=SUM(D5:D8)	=SUM(E5:E8)	=SUM(F5:F8)	

	Location	Salary	MS	Long	AVE
Location	=C5/$C$9	=D5/$D$9	=E5/$E$9	=F5/$F$9	=AVERAGE(C12:F12)
Salary	=C6/$C$9	=D6/$D$9	=E6/$E$9	=F6/$F$9	=AVERAGE(C13:F13)
MS	=C7/$C$9	=D7/$D$9	=E7/$E$9	=F7/$F$9	=AVERAGE(C14:F14)
Long	=C8/$C$9	=D8/$D$9	=E8/$E$9	=F8/$F$9	=AVERAGE(C15:F15)
				SUM	=SUM(G12:G15)

- In Excel, perform a pair-wise criteria evaluation for the relative importance of how you spend your awake time in:
  - Work
  - Learning (academic, advancement, etc.)
  - Recreation (sports, hobbies, games, TV, etc.)
  - Relationships (significant other, family, friends)
  - Devotion, Service, Reflection

Exercise

	Location	Salary	MS	Long	
Location	1	=1/C6	=1/C7	=1/C8	
Salary	5	1	=1/D7	=1/D8	
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SUM	=SUM(C5:C8)	=SUM(D5:D8)	=SUM(E5:E8)	=SUM(F5:F8)	

	Location	Salary	MS	Long	AVE
Location	=C5/$C$9	=D5/$D$9	=E5/$E$9	=F5/$F$9	=AVERAGE(C12:F12)
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MS	=C7/$C$9	=D7/$D$9	=E7/$E$9	=F7/$F$9	=AVERAGE(C14:F14)
Long	=C8/$C$9	=D8/$D$9	=E8/$E$9	=F8/$F$9	=AVERAGE(C15:F15)
				SUM	=SUM(G12:G15)

- Now tabulate how many weekly hours you actually spend on each; total should be about 896 (8 x 16 x 7):
  - Work
  - Learning (academic, advancement, etc.)
  - Recreation (sports, hobbies, games, TV, etc.)
  - Relationships (significant other, family, friends)
  - Devotion, Service, Reflection
- Compare the ratios with your priorities

Evaluate Alternatives

- The next step is to evaluate all the alternatives (jobs – A,B,C,D) on each objective
- For instance, if we take Location, and if we prefer to be in the northeast (and preferably Boston), and say the jobs are located in Pittsburgh, New York, Boston, and San Francisco respectively, then we might get the following matrix:

Location Scores

[i.e. B is twice as good as A in regard to location]

	A	B	C	D
A	1	1/2	1/3	5
B	2	1	1/2	7
C	3	2	1	9
D	1/5	1/7	1/9	1

Relative (normalized) Location Scores

	A	B	C	D	Average
A	.161	.137	.171	.227	.174
B	.322	.275	.257	.312	.293
C	.484	.549	.514	.409	.489
D	.032	.040	.057	.045	.044

In Excel:

18	**Location**						
19			**A**	**B**	**C**	**D**	
20		**A**	1.0000000	0.5000000	0.3333333	5.0000000	
21		**B**	2.0000000	1.0000000	0.5000000	7.0000000	
22		**C**	3.0000000	2.0000000	1.0000000	9.0000000	
23		**D**	0.2000000	0.1428571	0.1111111	1.0000000	
24		**SUM**	6.2000000	3.6428571	1.9444444	22.0000000	
25							
26			**A**	**B**	**C**	**D**	**AVE**
27		**A**	0.1612903	0.1372549	0.1714286	0.2272727	0.1743116
28		**B**	0.3225806	0.2745098	0.2571429	0.3181818	0.2931038
29		**C**	0.4838710	0.5490196	0.5142857	0.4090909	0.4890668
30		**D**	0.0322581	0.0392157	0.0571429	0.0454545	0.0435178

Relative Location Scores (con't)

- In words, of the the total "Location Value" available, Job C has about 50%, B has about 30%, A has about 17% and D has about 4%
- We now go through a similar process with Salary, amount of MS, and long term prospects
- Suppose the relative values for those objectives can be given as follows:

Relative Scores (average column) for Each Objective

	A	B	C	D
Location	.174	.293	.489	.044
Salary	.050	.444	.312	.194
MS	.210	.038	.354	.398
Long	.510	.012	.290	.188

	A	B	C	D	Average
A	.161	.137	.171	.227	.174
B	.322	.275	.257	.312	.293
C	.484	.549	.514	.409	.489
D	.032	.040	.057	.045	.044

Location Table

Results…

Job Table

	A	B	C	D
Location	.174	.293	.489	.044
Salary	.050	.444	.312	.194
MS	.210	.038	.354	.398
Long	.510	.012	.290	.188

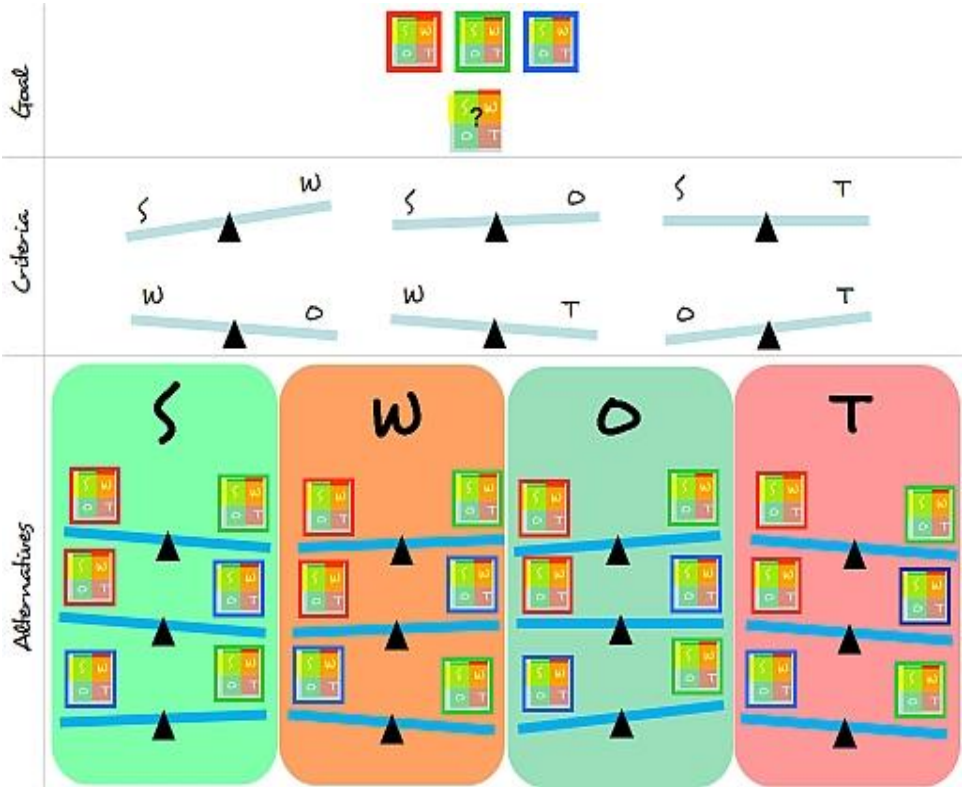
Criteria Table

	Location	Salary	MS	Long	Average
Location	.091	.102	.091	.059	.086
Salary	.455	.513	.545	.471	.496
MS	.273	.256	.273	.353	.289
Long	.182	.128	.091	.118	.130

- Recalling our overall weights for the objectives, we can now get a value for each job
  - The value for Acme Manufacturing (A) is:
  - .174*.086 + .05*.496 + .21*.289 + .51*.13 = .164
- Similarly, the value Bankers Bank (B) is .256
- The value for Creative Consultants (C) is **.335**
- The value for Dynamic Decision (D) is .238
- Creative Consultants it is the choice !

AHP Summary

- The Analytic Hierarchy Process is a method for formalizing decision making where there are a limited number of choices but each has a number of criteria and it is difficult to quantity some of those criteria



AHP Summary (con't)

- Note in this example, we did not collect any data (like miles from a preferred point or salary numbers)
- Instead, we use phrases like "much more important than" to extract the decision makers preferences

Table 3. **Computation of Hypothetical Sample Difference - AHP**

Analytic Hierarchy Process					
Subject	ALTERNATIVES	Stage 1 - (BASE)	Stage 2 - (AHP)	Difference	Diff Squared
1	Peotone	.35	.15	.20	.0400
	Bi-State	.40	.40	.00	.0000
	Gary	.25	.45	-.20	.0400
2	Peotone	.10	.40	-.30	.0900
	Bi-State	.25	.35	-.10	.0100
	Gary	.65	.25	.40	.1600
3	Peotone	.10	.25	-.15	.0225
	Bi-State	.10	.35	-.25	.0625
	Gary	.80	.40	.40	.1600
Mean of Differences Squared –>.1950					

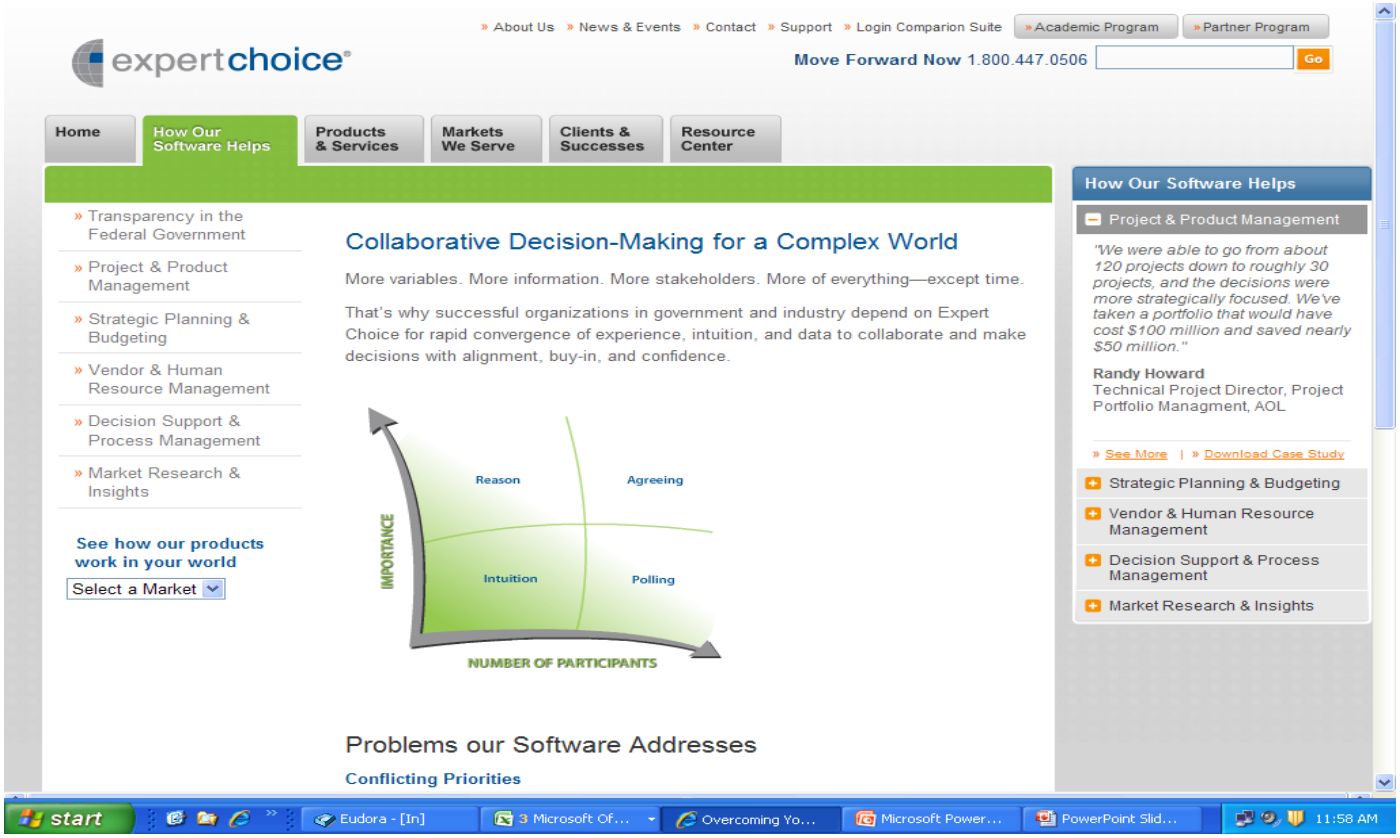
AHP Summary (con't)

- Despite the rather arbitrary aspects of the procedure, however, it does provide useful insight into the tradeoffs embedded in a complex decision making problem
- It also has the advantage of getting "buy-in" from the participants in determining the weights

AHP in Evaluating Business Initiatives or Projects

Software for AHP: Expert Choice

- No Excel-QM nor QM module for AHP



Homework

- Textbook online module 1
- Project 13 (in Excel)
  - Use AHP for your new car selection
  - Criteria (one level):
    - Price
    - Gas Mileage (MPG)
    - Safety
    - Capacity
  - Alternatives – pick 3 autos to evaluate