Management Science

Integer and Mixed Programming

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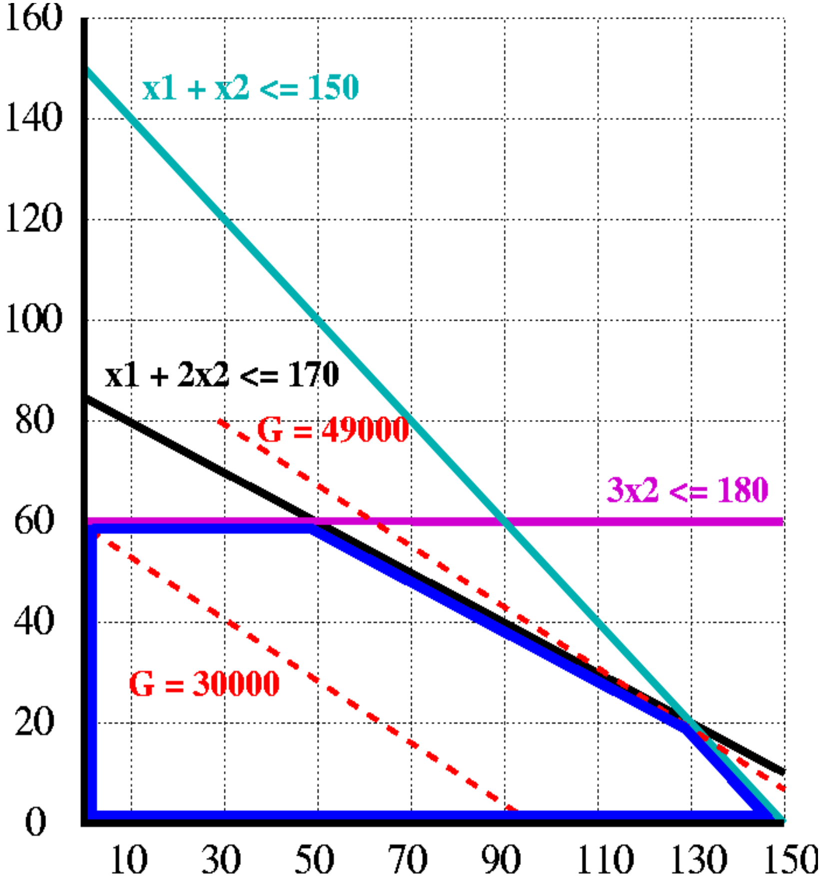
LP Problems Today

- Today the main difficulty is formulating the LP problem, not in solving it !!!



Packing Problem

- In this type of problem we have to pack a number of items into a number of physical or logical spaces
- We want to optimize the number of items or the value of the items packed into the spaces
- There are constraints on how much can be packed into the spaces based on characteristics of the “items” and characteristics of the “space”

Capacity of Plane

Space	Weight Capacity (tons)	Volume Capacity (cu ft)
Forward Hold	75	4000
Main Hold	150	10000
Aft Hold	50	7000

Available Cargo

[which should we take none of, which should we take all of]

Cargo	Tons	Cu ft/ton	$/ton
Memory Chips	200	60	800
NIC's	100	48.6	600
Monitors	500	4.1 (dense)	200 (cheap)
Mother Boards (w cpu)	50	240 (bulky)	2500 (expensive)

- What are the variables ?
- What is the objective function ?
- What are the constraints ?

- Do not look ahead !

Variables

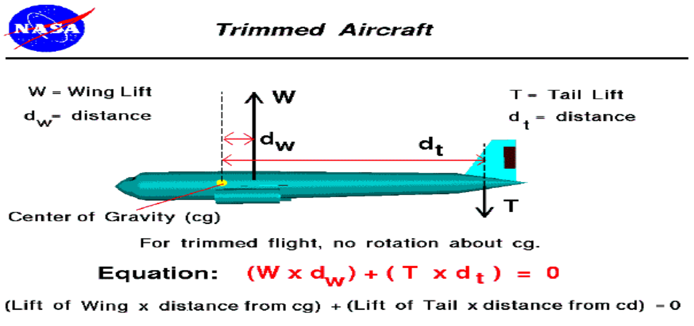
- Let:
  - W = tons of memory chips
  - X = tons of NIC's
  - Y = tons of monitors
  - Z = tons of mother boards

Objective Function

- Objective function to maximize:
  - Z = 800 W + 600 X + 200 Y + 2500 Z

Constraints

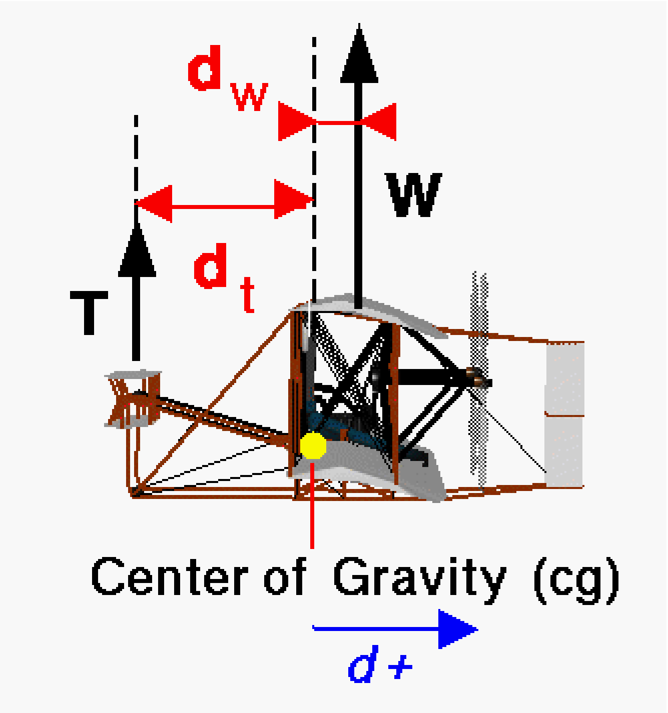
- Availability of cargo (supply)
- Weight of each hold
- Capacity (volume) of each hold
- What if we do not load the holds "evenly" by weight ?



- Do not look ahead !

Distribution Variables

- More variables:
  - WF = W in forward hold (chips in forward hold)
  - WC = W in main hold
  - WA = W in aft hold
  - Similarly for other items:
    - XF, XC, XA
    - YF, YC, YA
    - ZF, ZC, ZA



Distribution Constraints

- W = WF + WC + WA
- X = XF + XC + XA
- Y = YF + YC + YA
- Z = ZF + ZC + ZA

Trim Constraints

- The ratio of the weight of the cargo stored in each hold to that hold's maximum weight must be equal across all three holds !

  - (WF + XF + YF + ZF)/75 = (WC + XC + YC + ZC)/150
  - (WA + XA + YA + ZA)/50 = (WC + XC + YC + ZC)/150

Supply Constraints

- W <= 200 (chips)
- X <= 100 (NIC’s)
- Y <= 500 (monitors)
- Z <= 50 (boards)

Weight Constraints

- WF + XF + YF + ZF <=75 (forward)
- WC + XC + YC + ZC <= 150 (center)
- WA + XA + YA + ZA <= 50 (aft)

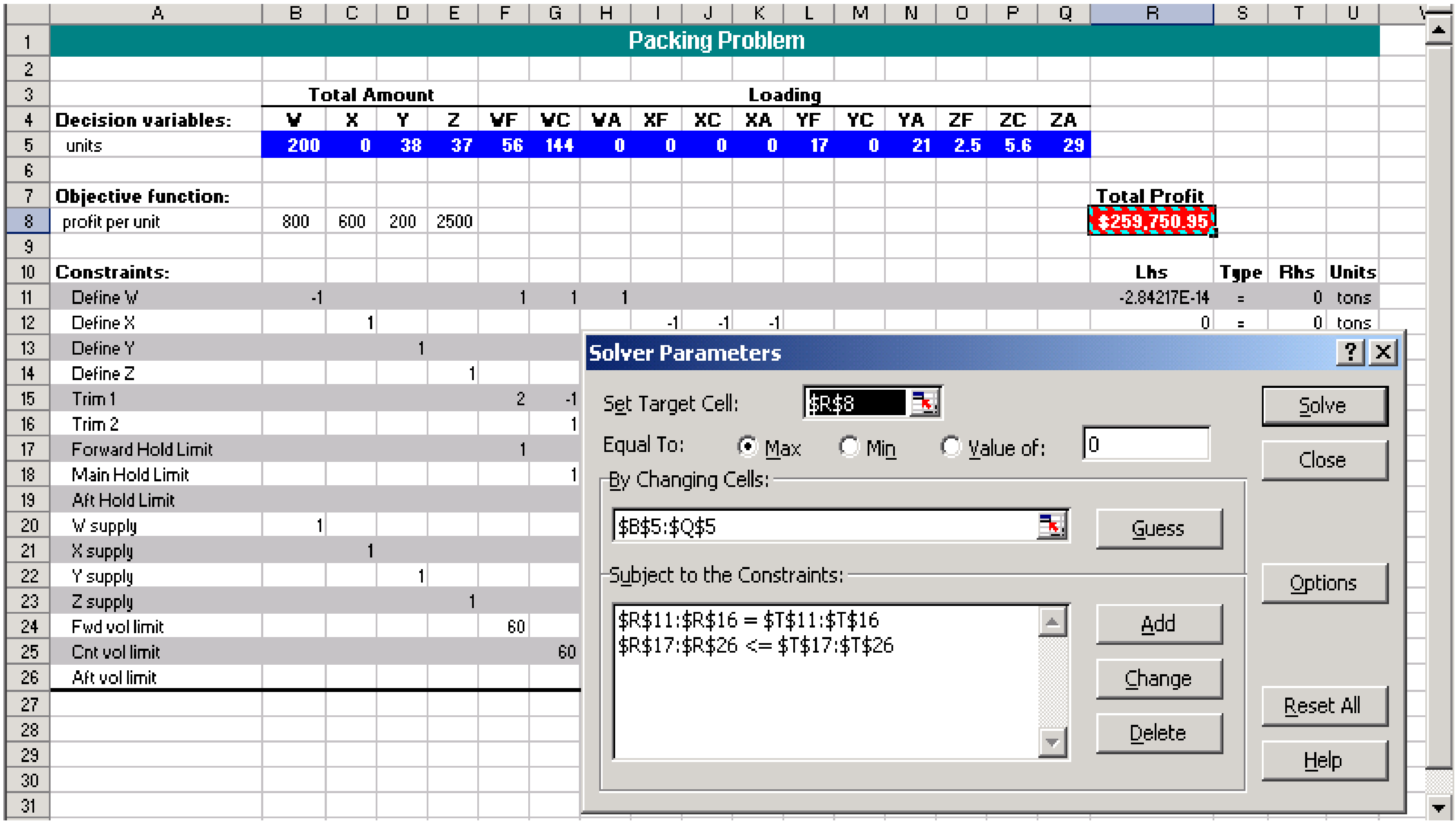
Volume Constraints

- 60 WF + 48.6 XF + 4.1 YF + 240 ZF <= 4000 (forward)
- 60 WC + 48.6 XC + 4.1 YC + 240 ZC <= 10000 (center)
- 60 WA + 48.6 XA + 4.1 YA + 240 ZA <= 7000 (aft)

Excel Solution

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Packing Problem																				
2																					
3		Total Amount				Loading															
4	**Decision variables:**	**W**	**X**	**Y**	**Z**	**WF**	**WC**	**WA**	**XF**	**XC**	**XA**	**YF**	**YC**	**YA**	**ZF**	**ZC**	**ZA**				
5	units	**200**	**0**	**38**	**37**	**56**	**144**	**0**	**0**	**0**	**0**	**17**	**0**	**21**	**2.5**	**5.6**	**29**				
6																					
7	**Objective function:**																	**Total Profit**			
8	profit per unit	800	600	200	2500													**$259,750.95**			
9																					
10	**Constraints:**																	**Lhs**	**Type**	**Rhs**	**Units**
11	Define W	-1				1	1	1										-2.84217E-14	=	0	tons
12	Define X		1						-1	-1	-1							0	=	0	tons
13	Define Y			1								-1	-1	-1				-7.10543E-15	=	0	tons
14	Define Z				1										-1	-1	-1	-7.10543E-15	=	0	tons
15	Trim 1					2	-1		2	-1		2	-1		2	-1		1.15463E-14	=	0	---
16	Trim 2						1	-3		1	-3		1	-3		1	-3	-1.42109E-14	=	0	---
17	Forward Hold Limit					1			1			1			1			75	<=	75	tons
18	Main Hold Limit						1			1			1			1		150	<=	150	tons
19	Aft Hold Limit							1			1			1			1	50	<=	50	tons
20	W supply	1																200	<=	200	tons
21	X supply		1															0	<=	100	tons
22	Y supply			1														38.15175922	<=	500	tons
23	Z supply				1													36.84824078	<=	50	tons
24	Fwd vol limit					60			48.6			4.1			240			4000	<=	4000	tons
25	Cnt vol limit						60			48.6			4.1			240		10000	<=	10000	tons
26	Aft vol limit							60			48.6			4.1			240	7000	<=	7000	tons

Solver
[no NIC's, all the memory chips]



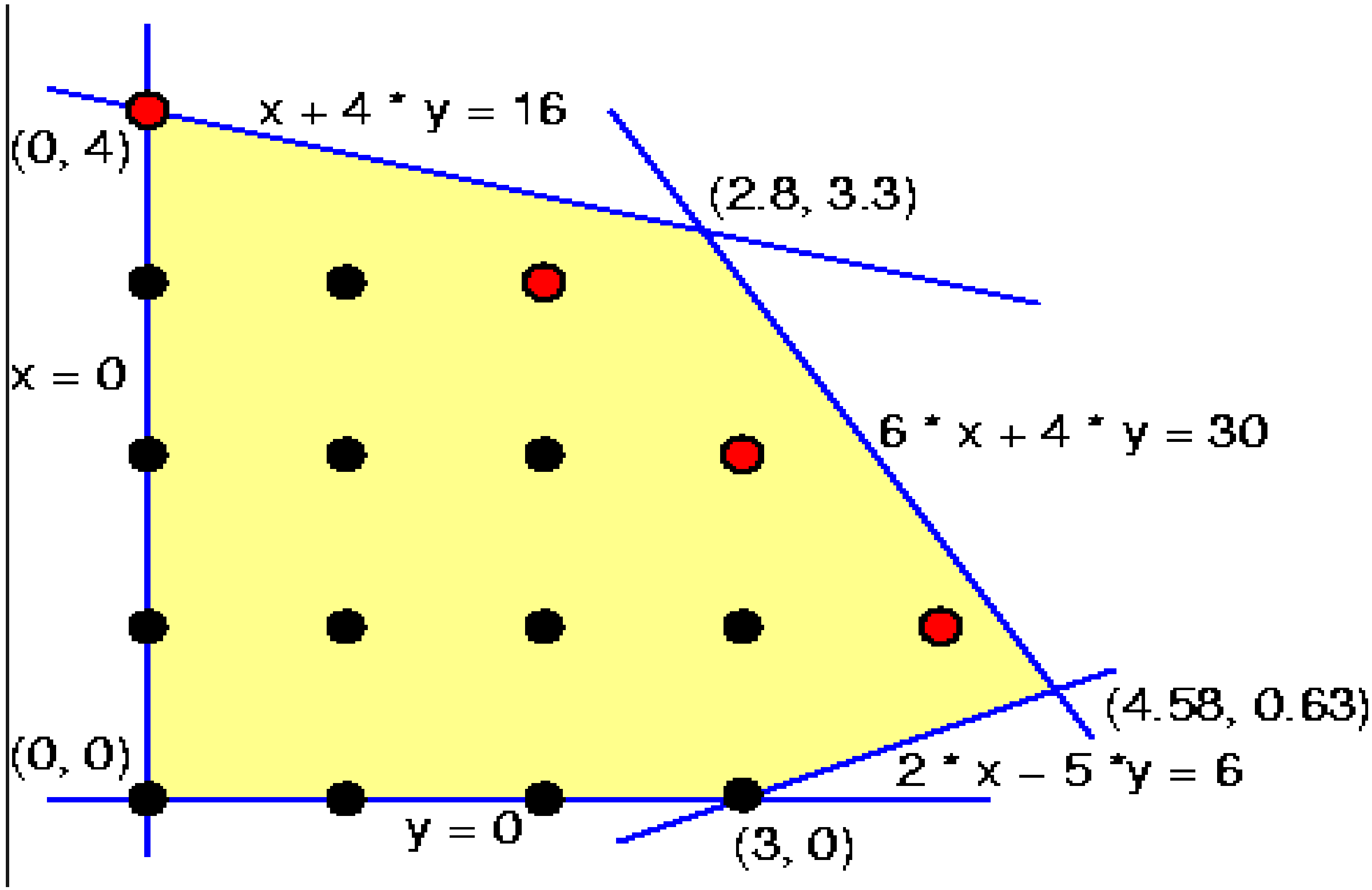
Integer Programming

- An integer programming model is one where one or more of the decision variables has to take on an integer value in the final solution
- There are three types of integer programming problems
  1. Pure integer programming where all variables have integer values
  2. Mixed-integer programming where some but not all of the variables will have integer values
  3. Zero-one integer (binary programming) are special cases in which all the decision variables must have integer solution values of 0 or 1

Integer Programming (con’t)

- Solving an integer programming problem is much more difficult than solving an LP problem
- Even the fastest computers can take an excessively long time to solve big integer programming problems
- There are a number of methods for solving integer programming problems, and a common technique used to solve integer programming problems is the branch and bound method
- There may be multiple solutions to integer programming problems

Integer Programming (con't)



Example of Integer Programming

- A company produces two products popular with home renovators: old-fashioned chandeliers and ceiling fans
- Both the chandeliers and fans require a two-step production process involving wiring and assembly
- It takes about 2 hours to wire each chandelier and 3 hours to wire a ceiling fan
- Final assembly of the chandeliers and fans requires 6 and 5 hours respectively
- The production capability is such that only 12 hours of wiring time and 30 hours of assembly time are available

Example of Integer Programming (con't)

- Each chandelier produced nets the firm $7 and each fan $6
- What are the variables ?
- What are the constraints ?

- Do not look ahead !

Example of Integer Programming (con't)

- This "production mix" decision can be formulated using LP as follows:

Maximize profit = $\$7X_1 + \$6X_2$

subject to

$$2X_1 + 3X_2 \le 12 \quad \text{(wiring hours)}$$
$$6X_1 + 5X_2 \le 30 \quad \text{(assembly hours)}$$
$$X_1, X_2 \ge 0 \quad \text{(nonnegative)}$$

where

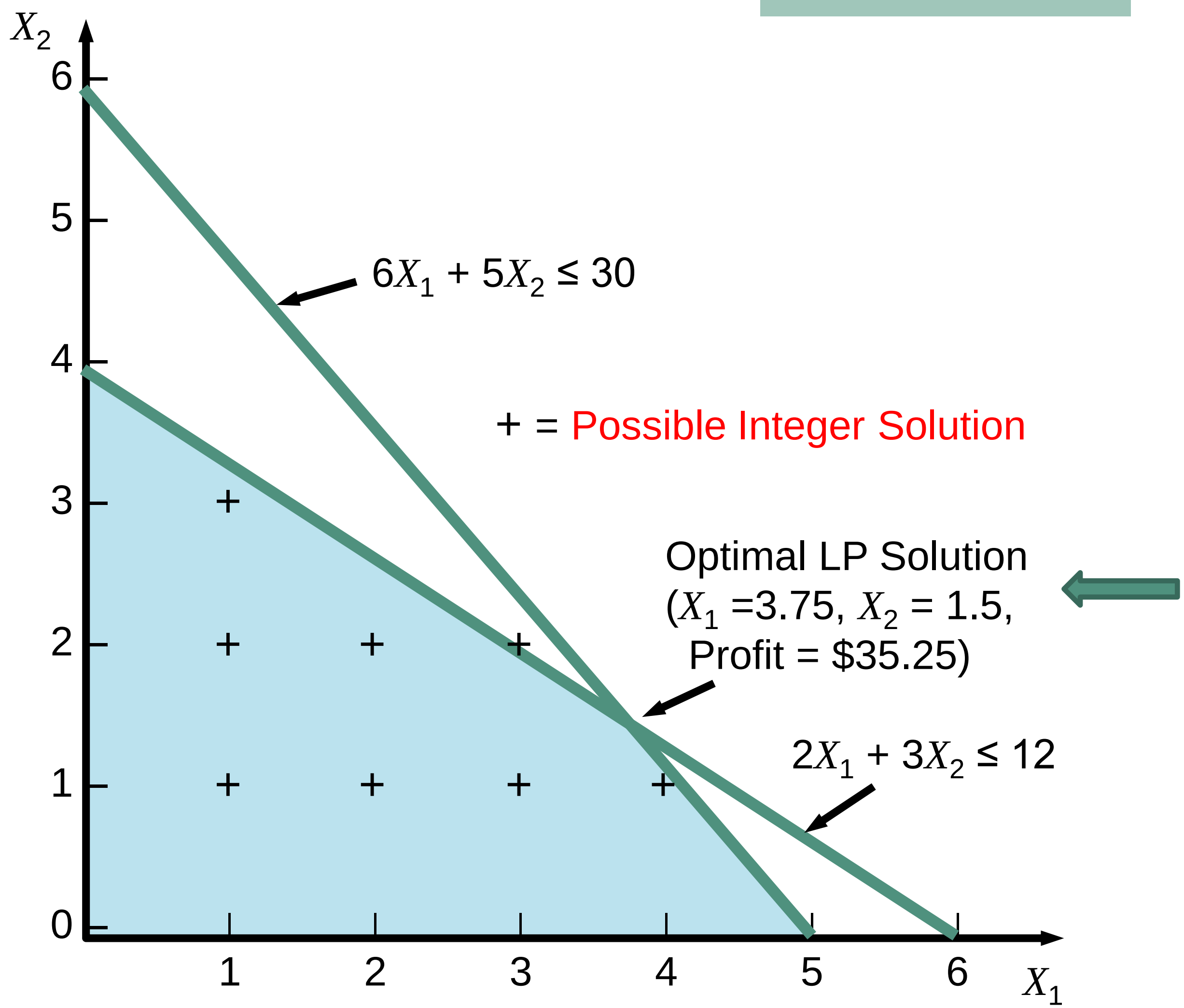
X_1 = number of chandeliers produced, must be an integer

X_2 = number of ceiling fans produced, must be an integer

- How could we approach solving this problem ?

- Do not look ahead !

Example of Integer Programming (con't)



Example of Integer Programming (con't)

- A first attempt at solving may be to round the optimal values to $X_1 = 4$ and $X_2 = 2$
- However, this is outside of the feasible area
- Rounding X_2 down to 1 gives a feasible solution, but it may not be *optimal*
- This could be solved using an *enumeration* method
- Enumeration is generally not feasible for large problems

Enumeration For Integer Programming

- Integer solutions

CHANDELIERS (X_1)	CEILING FANS (X_2)	PROFIT ($\$7X_1 + \$6X_2$)
0	0	$0
1	0	7
2	0	14
3	0	21
4	0	28
5	0	35 ← Optimal solution to integer programming problem
0	1	6
1	1	13
2	1	20
3	1	27
4	1	34
0	2	12
1	2	19
2	2	26
3	2	33
0	3	18
1	3	25
0	4	24

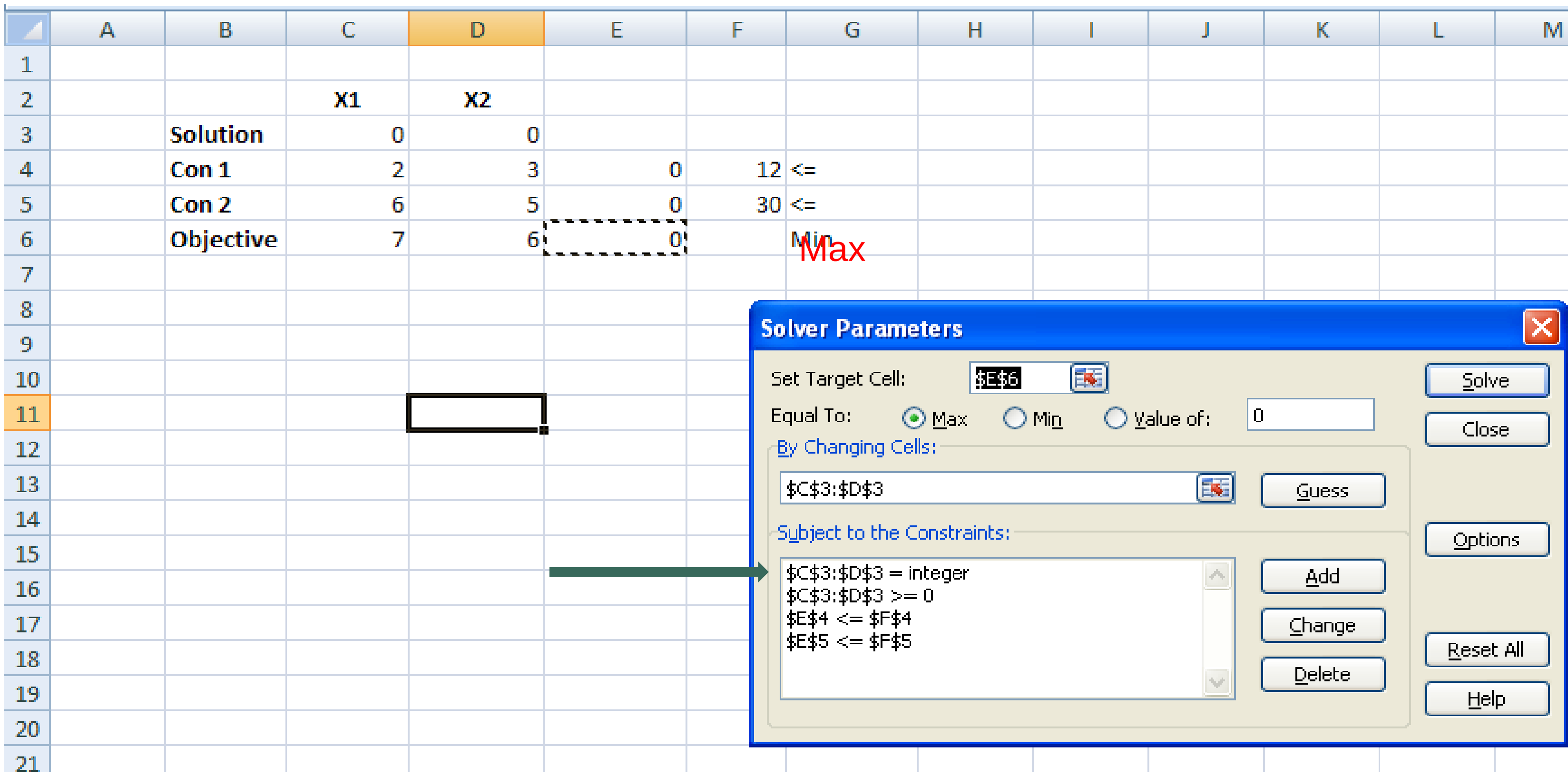
Example of Integer Programming (con't)

- The optimal integer solution of $X_1 = 5$, $X_2 = 0$ gives a profit of $35
- The optimal integer solution (35) is less than the optimal LP solution (35.25)
- An integer solution can *never* be better than the LP solution and is *usually* a lesser solution

Example of Integer Programming (con't)

	A	B	C	D	E	F	G
1							
2			**X1**	**X2**			
3		**Solution**	0	0			
4		**Con 1**	2	3	=$C$3*C4 + $D$3*D4	12	<=
5		**Con 2**	6	5	=$C$3*C5 + $D$3*D5	30	<=
6		**Objective**	7	6	=$C$3*C6 + $D$3*D6		Min MAX
7							
8							
9							

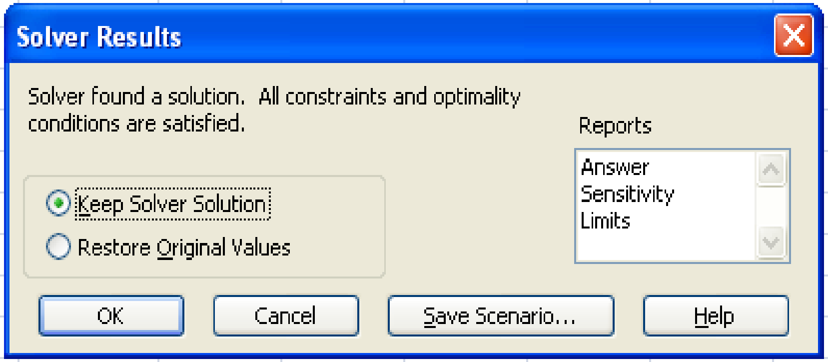
Example of Integer Programming (con’t)



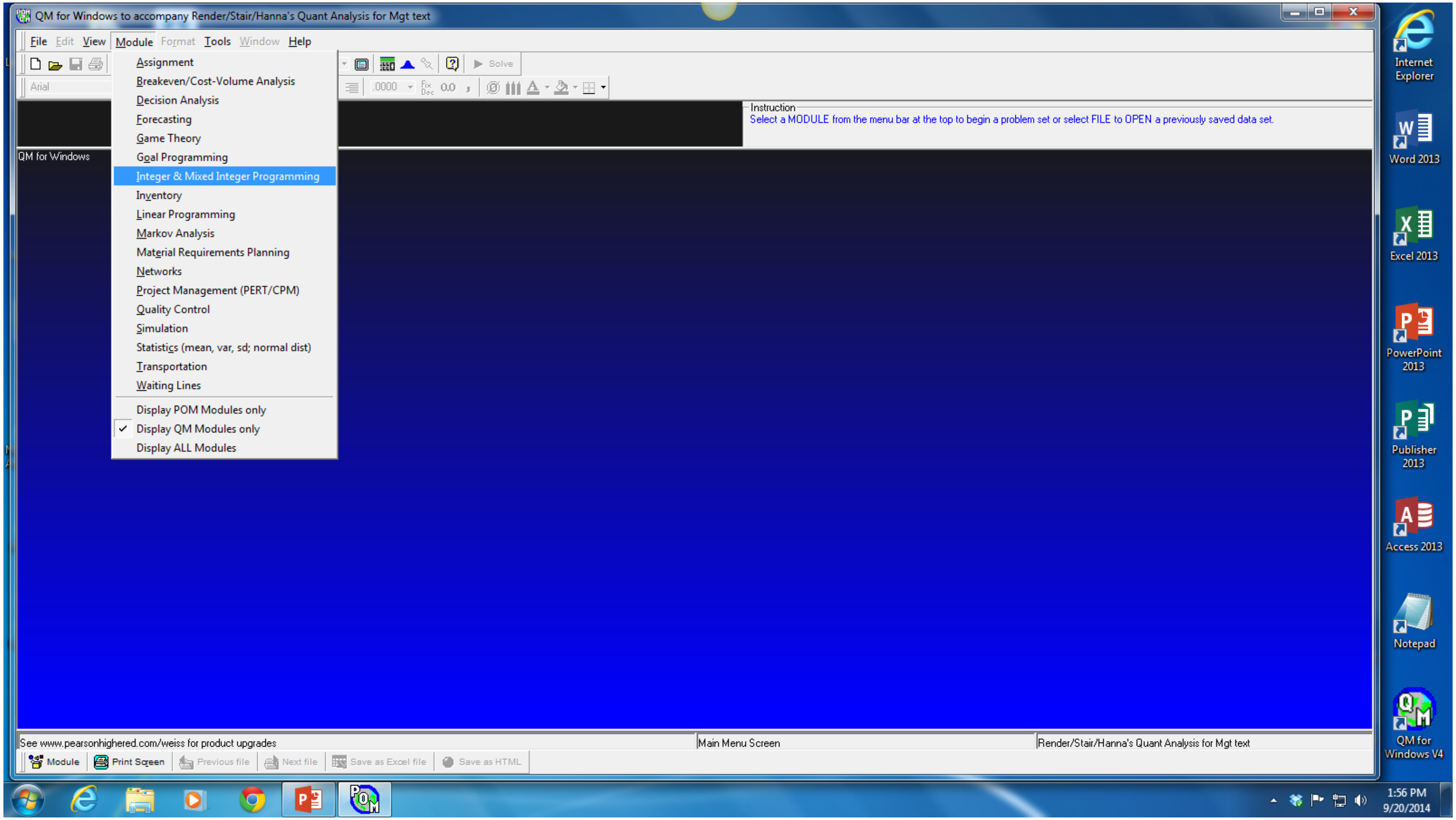
For integer problems, one still chooses the “simplex” method in Excel, Excel will automatically switch over to the “branch and bound” method

Example of Integer Programming (con't)

	X1	X2			
Solution	5	0			
Con 1	2	3	10	12	<=
Con 2	6	5	30	30	<=
Objective	7	6	35		Min



Using QM…


QM for Windows to accompany Render/Stair/Hanna's Quant Analysis for Mgt text
File Edit View Module Format Tools Window Help
Solve
Instruction
Select a MODULE from the menu bar at the top to begin a problem set or select FILE to OPEN a previously saved data set.
QM for Windows
Assignment
Breakeven/Cost-Volume Analysis
Decision Analysis
Forecasting
Game Theory
Goal Programming
Integer & Mixed Integer Programming
Inventory
Linear Programming
Markov Analysis
Material Requirements Planning
Networks
Project Management (PERT/CPM)
Quality Control
Simulation
Statistics (mean, var, sd; normal dist)
Transportation
Waiting Lines
Display POM Modules only
Display QM Modules only
Display ALL Modules
See www.pearsonhighered.com/weiss for product upgrades
Main Menu Screen
Render/Stair/Hanna's Quant Analysis for Mgt text
Module
Print Screen
Previous file
Next file
Save as Excel file
Save as HTML
Internet Explorer
Word 2013
Excel 2013
PowerPoint 2013
Publisher 2013
Access 2013
Notepad
QM for Windows V4
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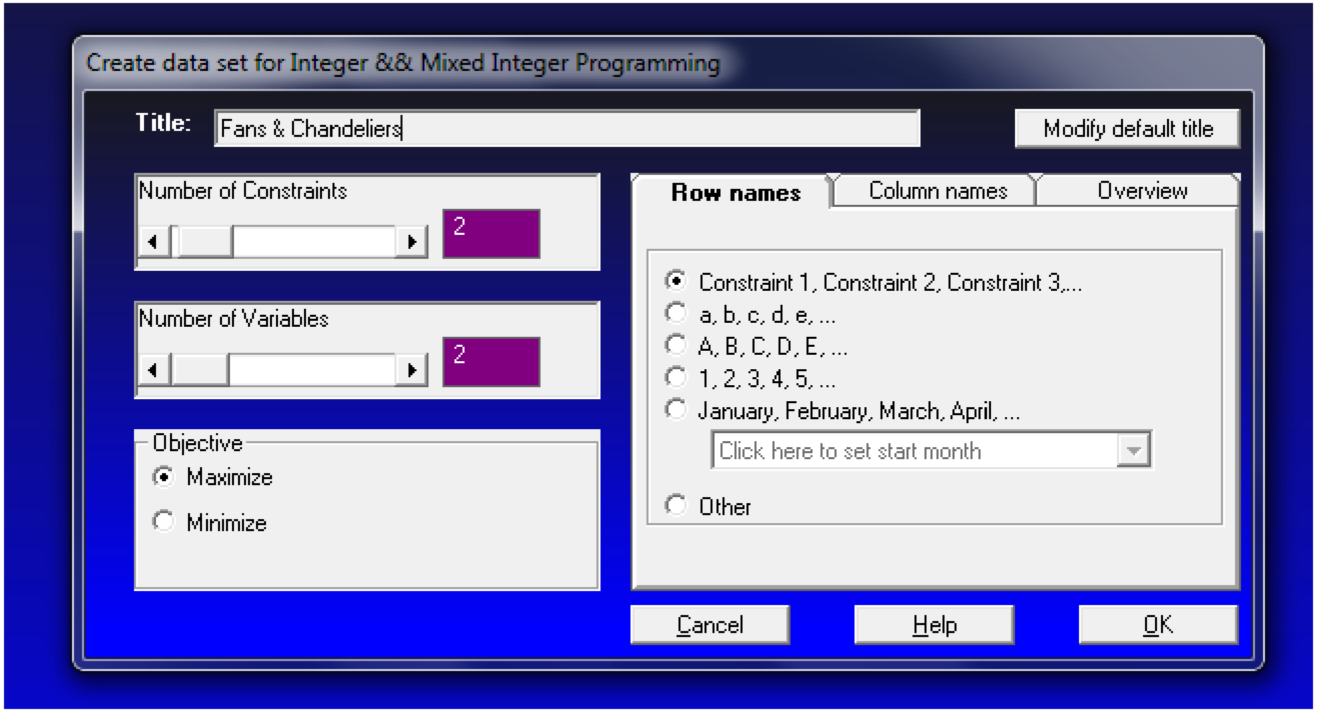
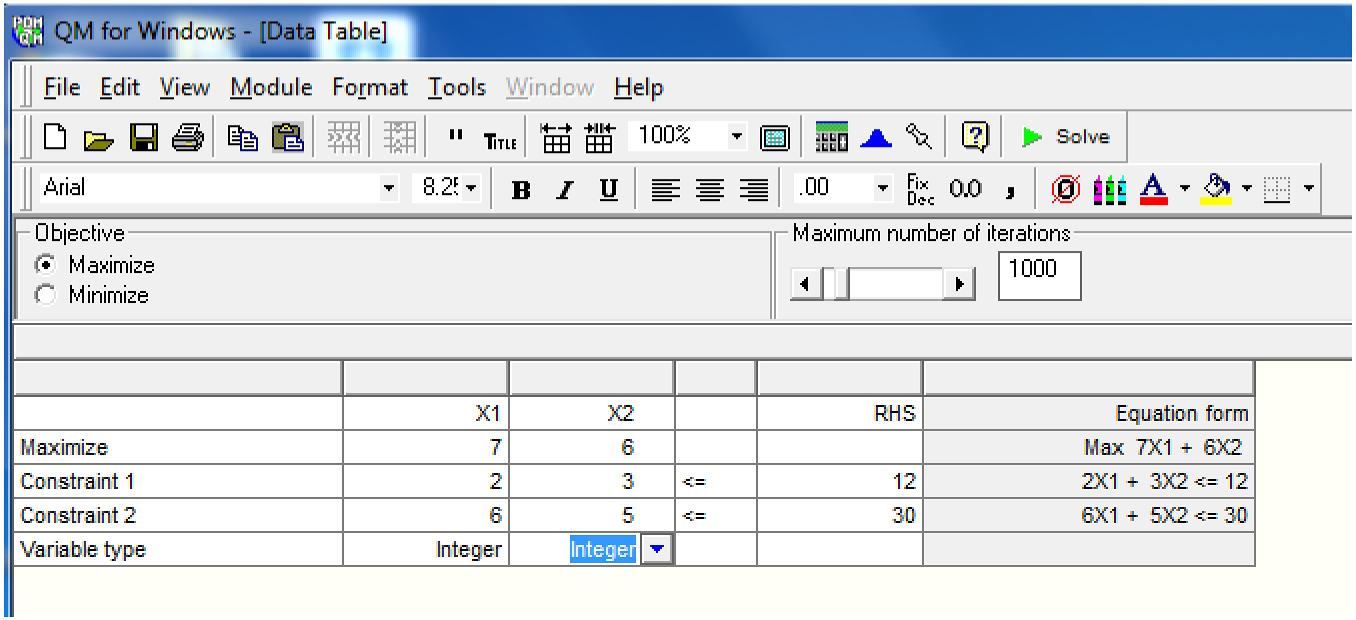
Using QM…

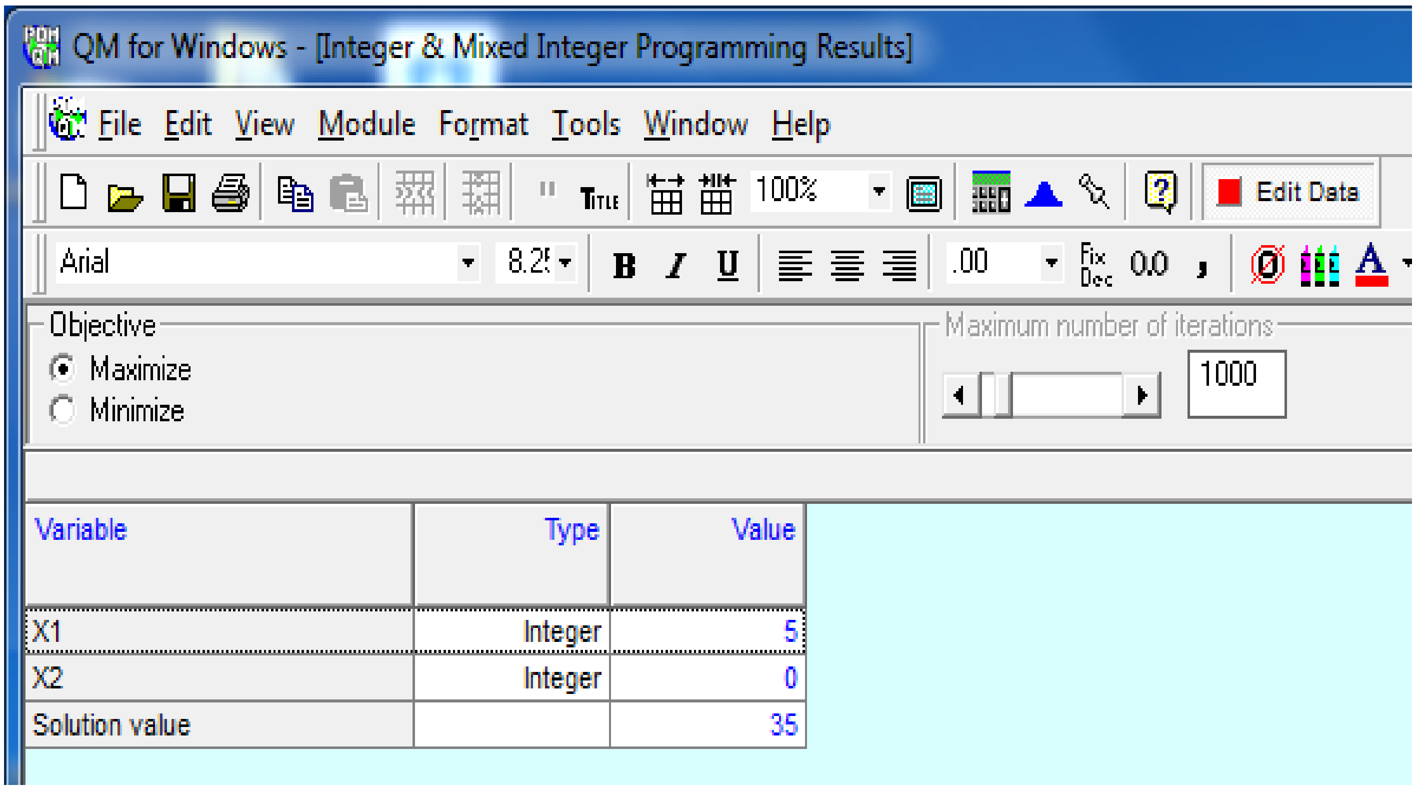


Create data set for Integer && Mixed Integer Programming
Title:
Fans & Chandeliers
Modify default title
Number of Constraints
2
Number of Variables
2
Objective
Maximize
Minimize
Row names
Column names
Overview
Constraint 1, Constraint 2, Constraint 3,...
a, b, c, d, e, ...
A, B, C, D, E, ...
1, 2, 3, 4, 5, ...
January, February, March, April, ...
Click here to set start month
Other
Cancel
Help
OK

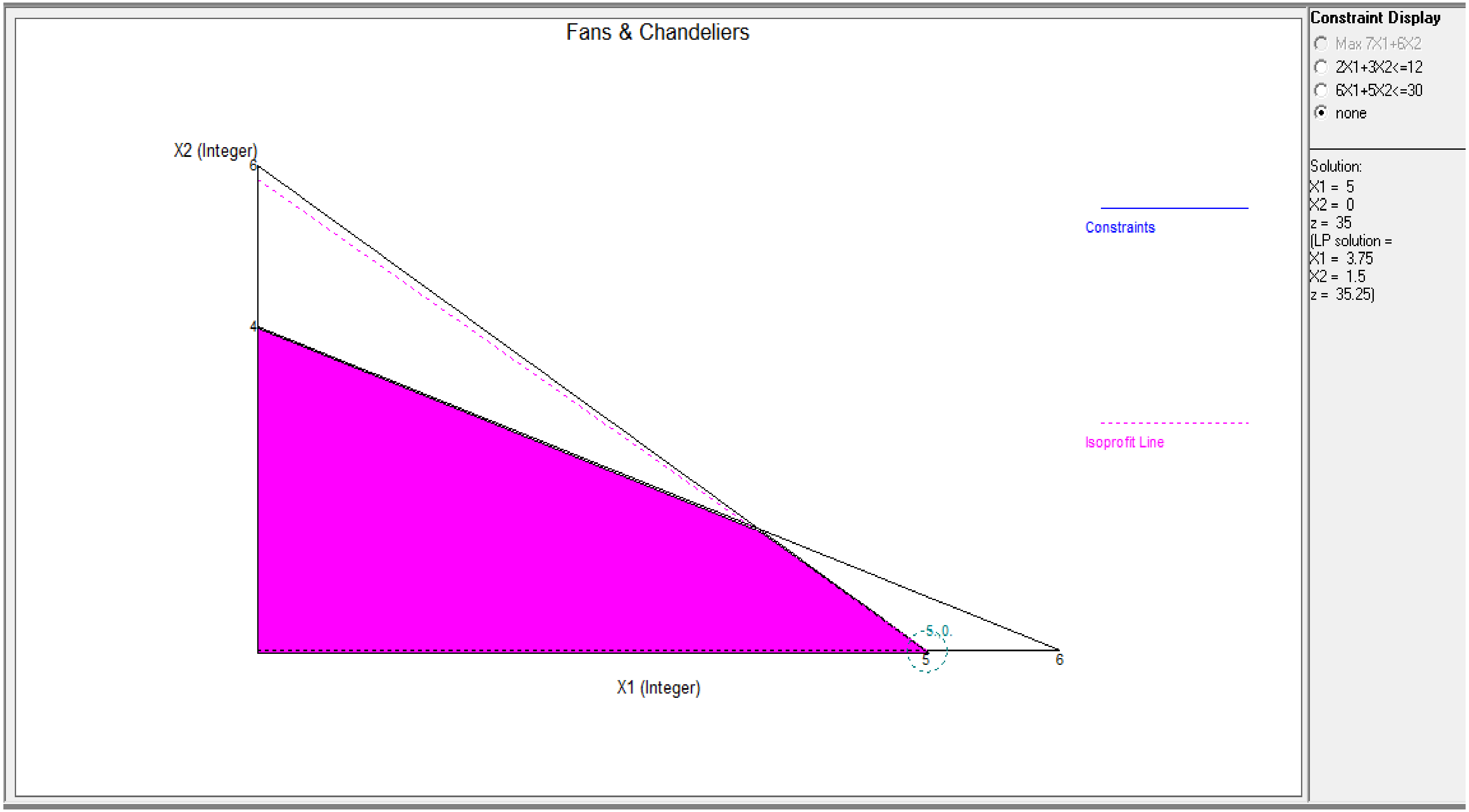
Using QM…


QM for Windows - [Data Table]
File Edit View Module Format Tools Window Help
100%
Solve
Arial
8.25
.00
Fix Dec
0.0
Objective
Maximize
Minimize
Maximum number of iterations
1000
X1
X2
RHS
Equation form
Maximize 7 6
Max 7X1 + 6X2
Constraint 1 2 3 <= 12
2X1 + 3X2 <= 12
Constraint 2 6 5 <= 30
6X1 + 5X2 <= 30
Variable type Integer Integer

Using QM…


QM for Windows - [Integer & Mixed Integer Programming Results]
File Edit View Module Format Tools Window Help
100%
Edit Data
Arial
.00
Objective
Maximize
Minimize
Maximum number of iterations
1000
Variable
Type
Value
X1
Integer
5
X2
Integer
0
Solution value
35

Using QM…

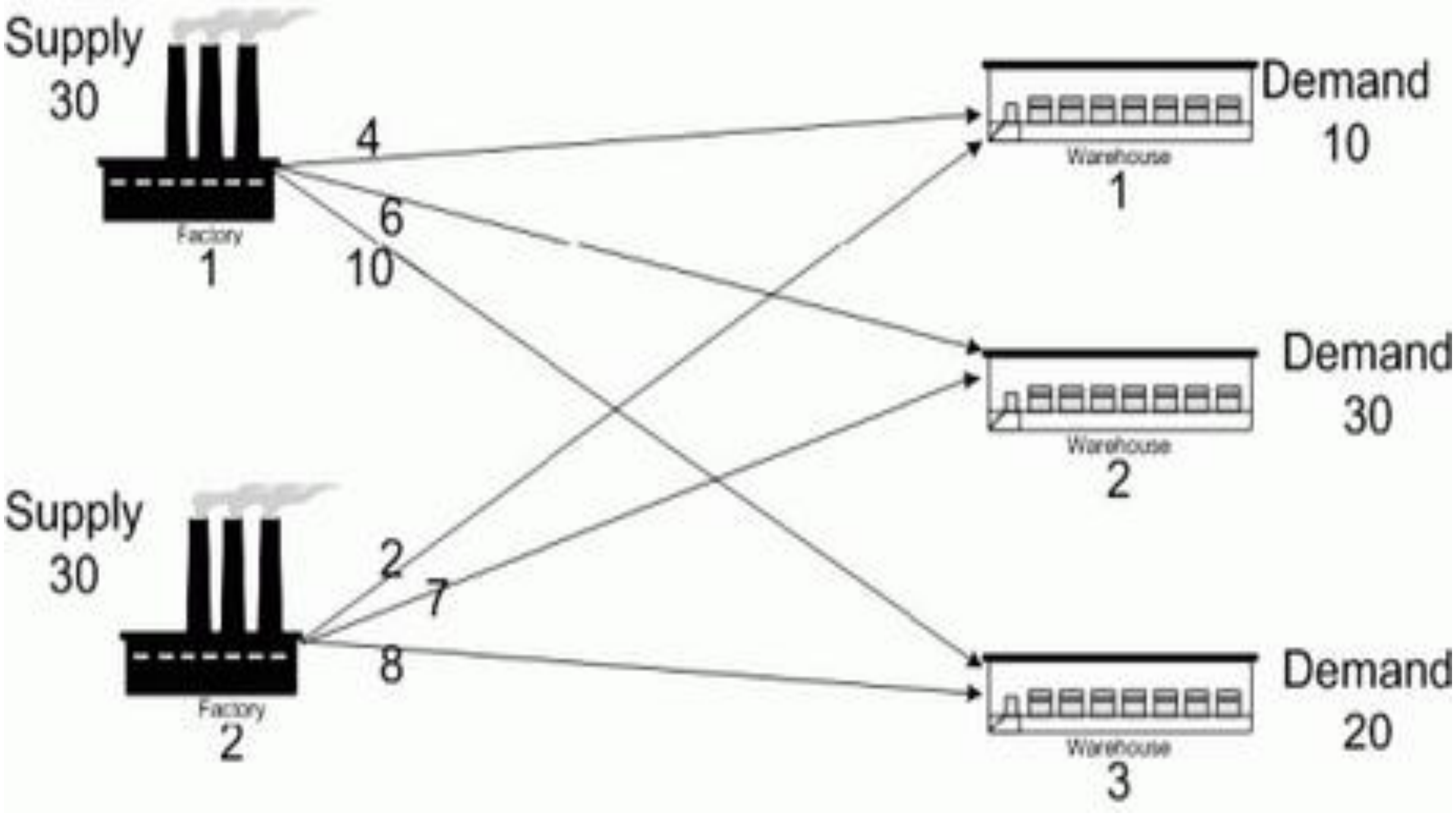


Fans & Chandeliers
X2 (Integer)
6
4
5
6
5, 0
X1 (Integer)
Constraints
Isoprofit Line
Constraint Display
Max 7X1+6X2
2X1+3X2<=12
6X1+5X2<=30
none
Solution:
X1 = 5
X2 = 0
z = 35
(LP solution =
X1 = 3.75
X2 = 1.5
z = 35.25)

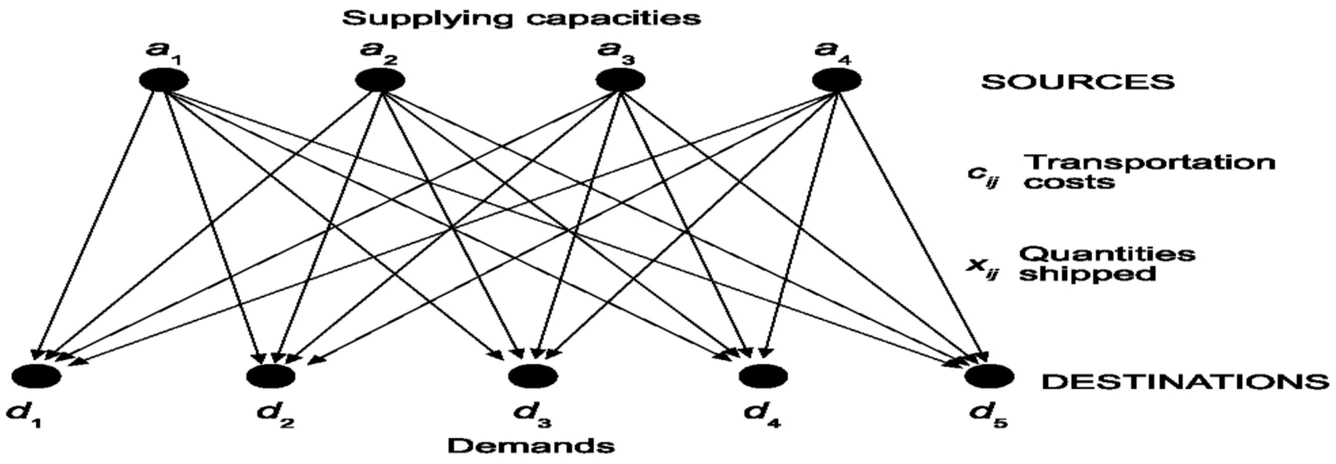
“Transportation” Problems

- There are a number of special management problems that can be solved via integer linear programming
- One common problem is that of the “transportation problem”
- Here we have a number of plants (or warehouses) that can produce (or ship) products subject to supply constraints and we have a number of consumers (or customers) that have certain demands for products
- There is a cost associated with shipping product from the warehouses to the customers

"Transportation" Problems (con't)



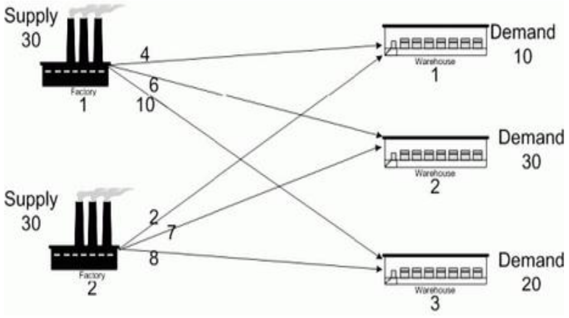
- What are the variables ?
- What is the objective function ?
- What are the constraints ?



- Do not look ahead !

"Transportation" Problems (con't)

- Let:
  - X_{ij} = number of units shipped from source *i* to destination *j* (*integers*)
  - c_{ij} = cost of one unit from source *i* to destination *j*
  - s_i = supply at source *i*
  - d_j = demand at destination *j*



“Transportation” Problems (con’t)

Minimize cost = $$\sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \leq s_i$$ *i = 1, 2,…, m [supply]*

$$\sum_{i=1}^{m} x_{ij} \leq d_j$$ *j = 1, 2, …, n [demand]*

$x_{ij} \geq 0$ *for all i and j, and are integers*

Transportation Matrix in Excel

[showing demand, supply, and transport costs]

Microsoft Excel - transportation.xls

		Customers			
	FROM\TO	Birmingham	New Orleans	Jackson	SUPPLY
Warehouses	Memphis	5	4	3	100
	Dallas	8	4	3	300
	Atlanta	9	7	5	300
	DEMAND	300	200	200	700/700

Cost from Dallas to Jackson

What are the cells for the decision variables ?

Set Up Shipment Table and Cost Function

Microsoft Excel - transportation.xls

D20 =SUMPRODUCT(C6:E8,C15:E17)

Problem - Transportation Parameters

		Customers			
	FROM\TO	Birmingham	New Orleans	Jackson	SUPPLY
Warehouses	Memphis	5	4	3	100
	Dallas	8	4	3	300
	Atlanta	9	7	5	300
	DEMAND	300	200	200	700/700

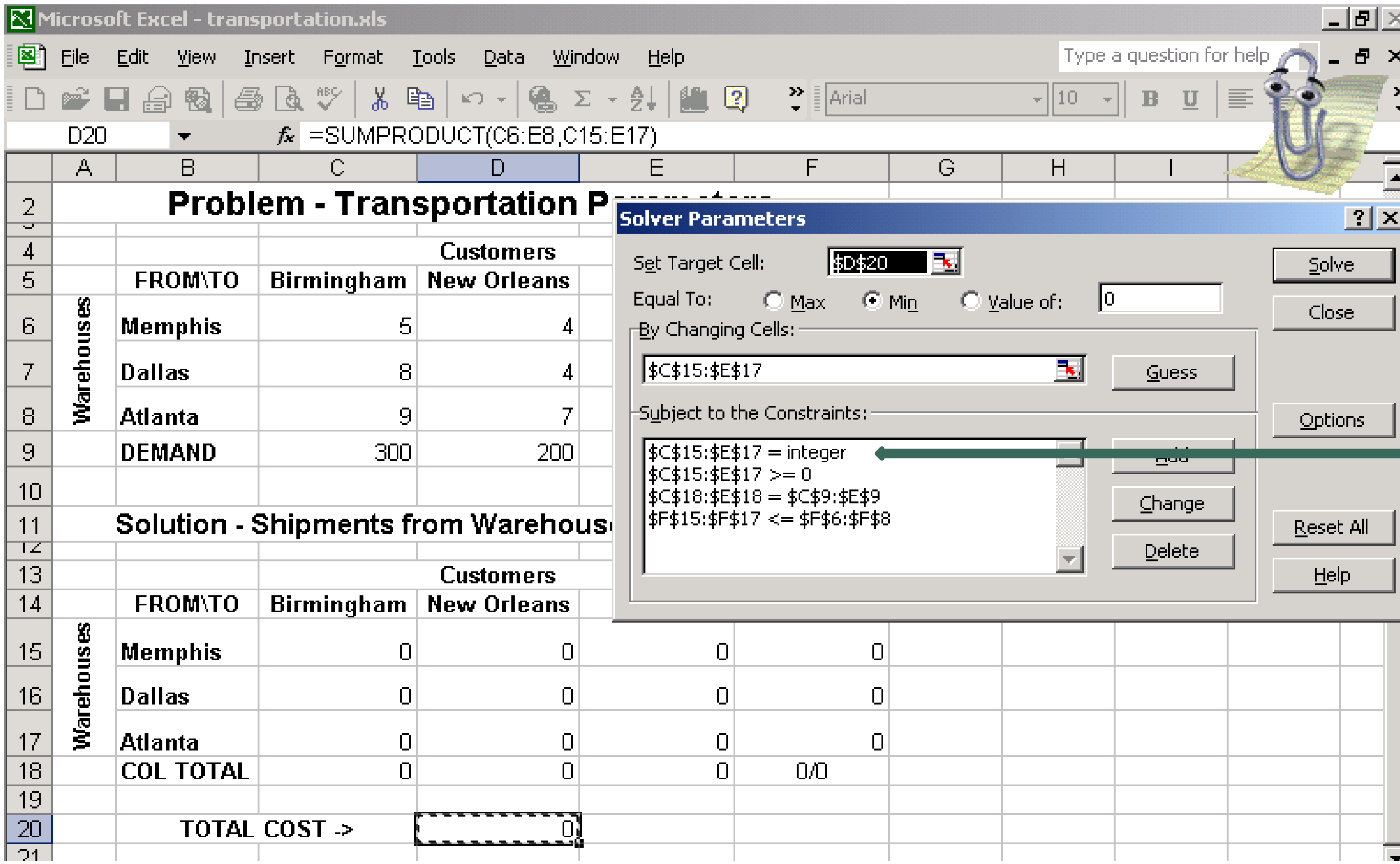
Solution - Shipments from Warehouses to Customers

		Customers			
	FROM\TO	Birmingham	New Orleans	Jackson	ROW TOTAL
Warehouses	Memphis	0	0	0	0
	Dallas	0	0	0	0
	Atlanta	0	0	0	0
	COL TOTAL	0	0	0	0/0
	TOTAL COST ->		0		

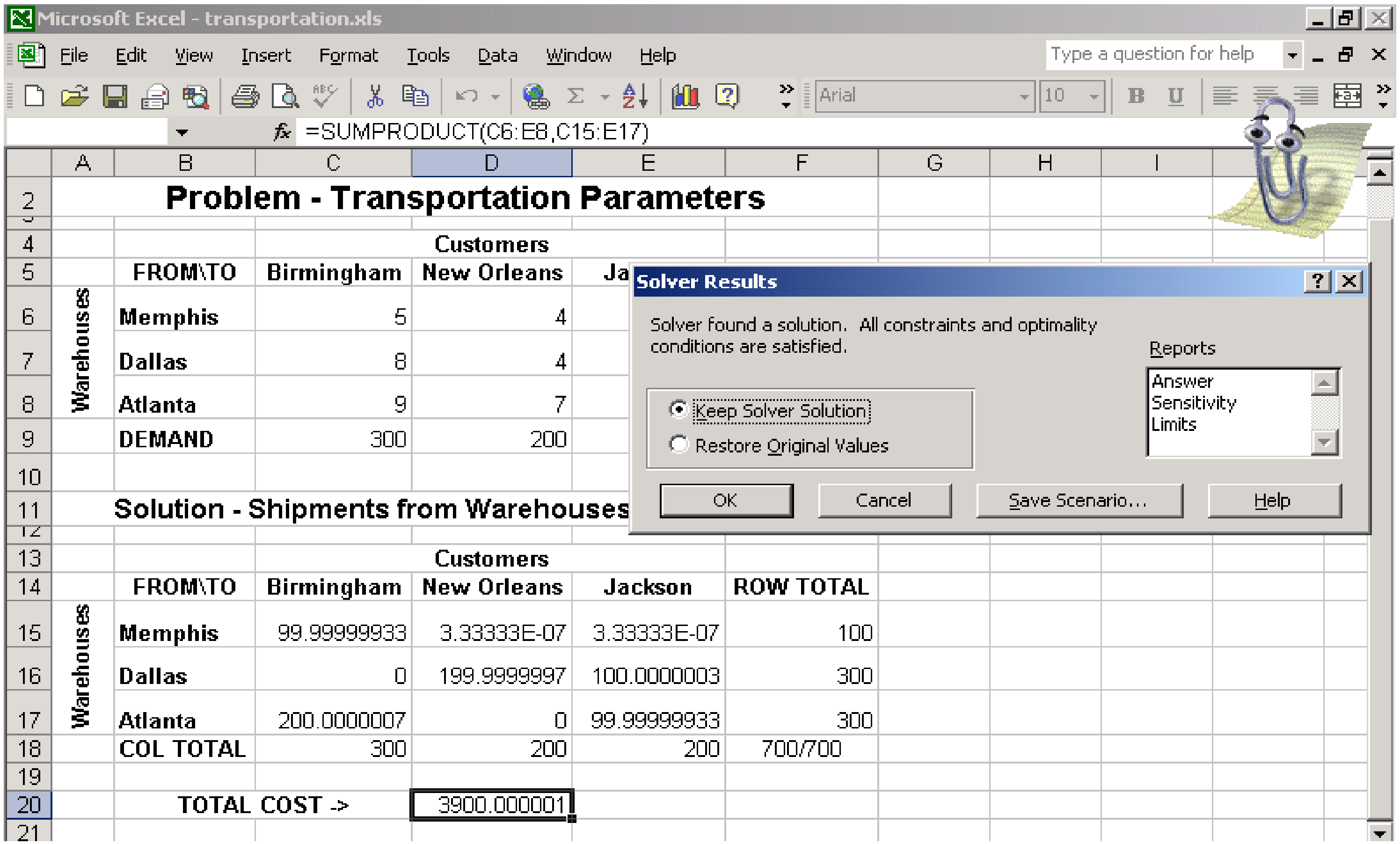
- Target is D20 (minimize)
- Manipulate c15:e17
- Subject to:
  - Col totals equal demand
  - Row totals <= supply
  - Shipments are integers and +

Need another matrix for decision variables.

Parameters for Excel Solver



Excel Solution



Microsoft Excel - transportation.xls

=SUMPRODUCT(C6:E8,C15:E17)

Problem - Transportation Parameters

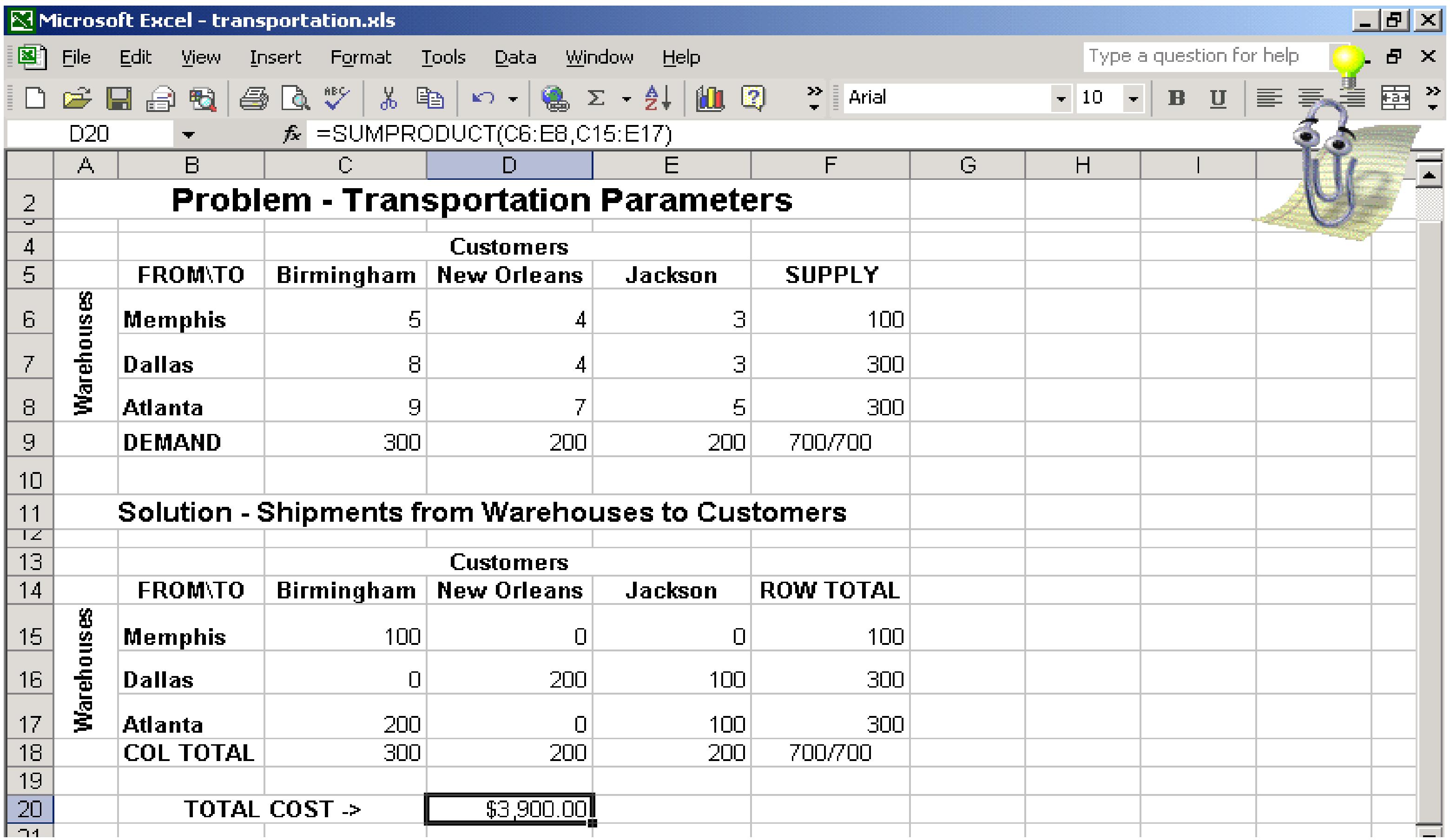
		Customers		
	FROM\TO	Birmingham	New Orleans	Ja
Warehouses	Memphis	5	4	
	Dallas	8	4	
	Atlanta	9	7	
	DEMAND	300	200	

Solution - Shipments from Warehouses

		Customers			
	FROM\TO	Birmingham	New Orleans	Jackson	ROW TOTAL
Warehouses	Memphis	99.99999933	3.33333E-07	3.33333E-07	100
	Dallas	0	199.9999997	100.0000003	300
	Atlanta	200.0000007	0	99.99999933	300
	COL TOTAL	300	200	200	700/700

TOTAL COST -> 3900.000001

After Formatting Cells



Problem - Transportation Parameters

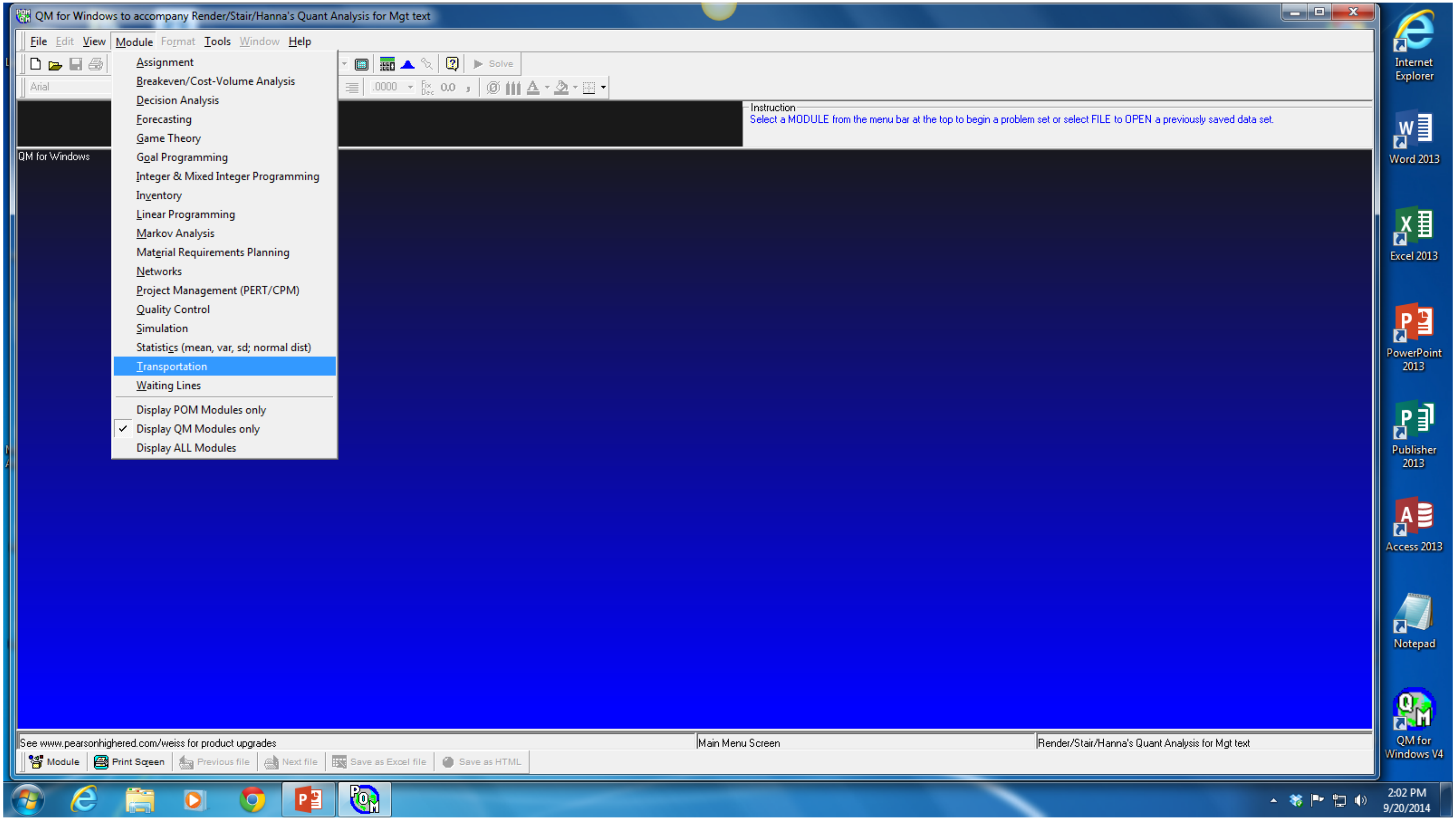
		Customers			
	FROM\TO	Birmingham	New Orleans	Jackson	SUPPLY
Warehouses	Memphis	5	4	3	100
	Dallas	8	4	3	300
	Atlanta	9	7	5	300
	DEMAND	300	200	200	700/700

Solution - Shipments from Warehouses to Customers

		Customers			
	FROM\TO	Birmingham	New Orleans	Jackson	ROW TOTAL
Warehouses	Memphis	100	0	0	100
	Dallas	0	200	100	300
	Atlanta	200	0	100	300
	COL TOTAL	300	200	200	700/700

TOTAL COST -> $3,900.00

Using QM…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Waiting Lines
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Select a MODULE from the menu bar at the top to begin a problem set or select FILE to OPEN a previously saved data set.
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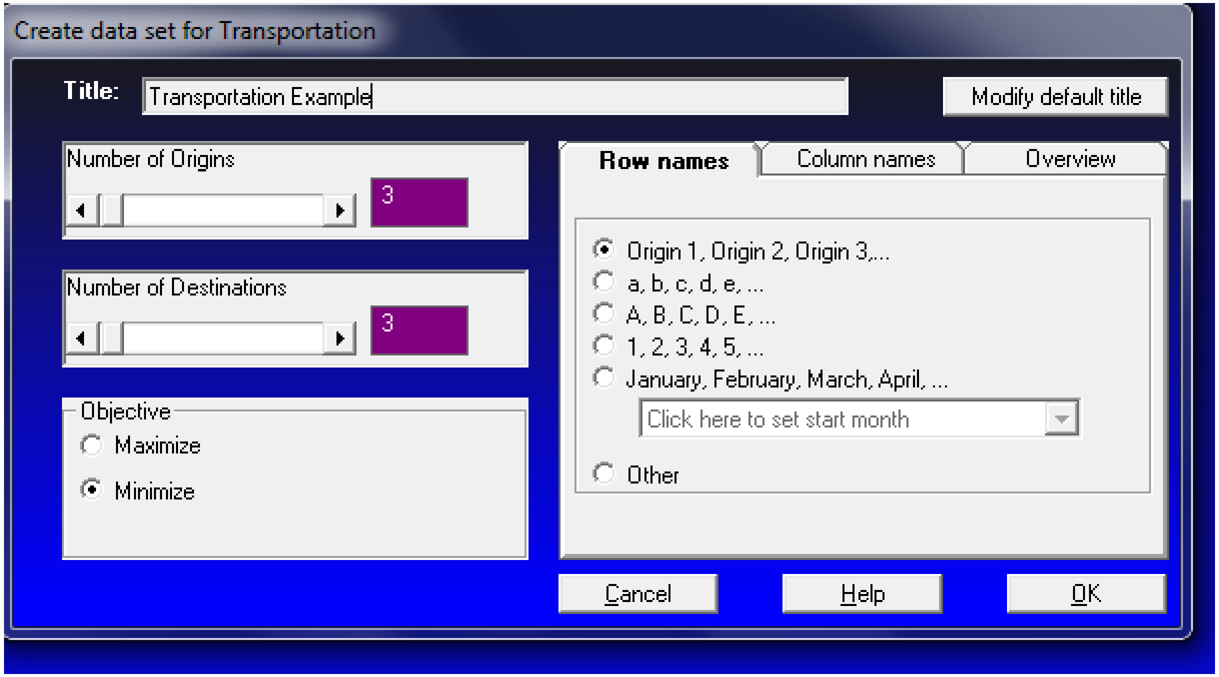
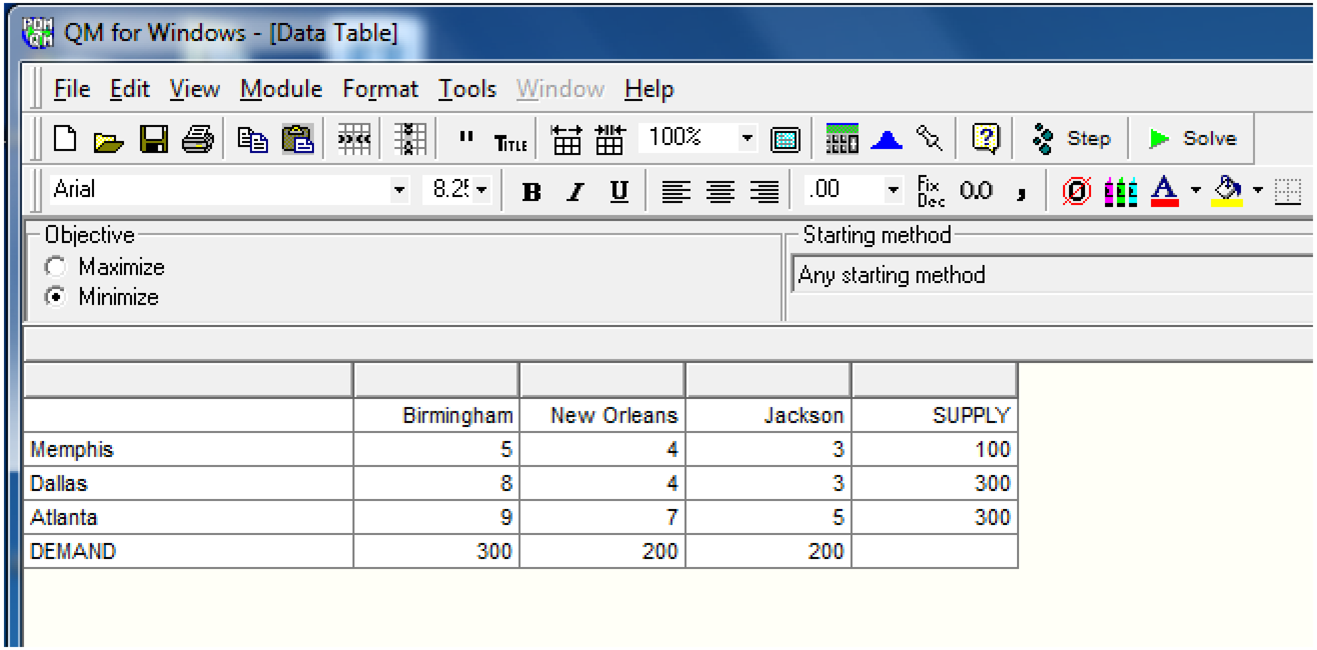
See www.pearsonhighered.com/weiss for product upgrades
Main Menu Screen
Render/Stair/Hanna's Quant Analysis for Mgt text
Module
Print Screen
Previous file
Next file
Save as Excel file
Save as HTML
Internet Explorer
Word 2013
Excel 2013
PowerPoint 2013
Publisher 2013
Access 2013
Notepad
QM for Windows V4
2:02 PM
9/20/2014

Using QM…

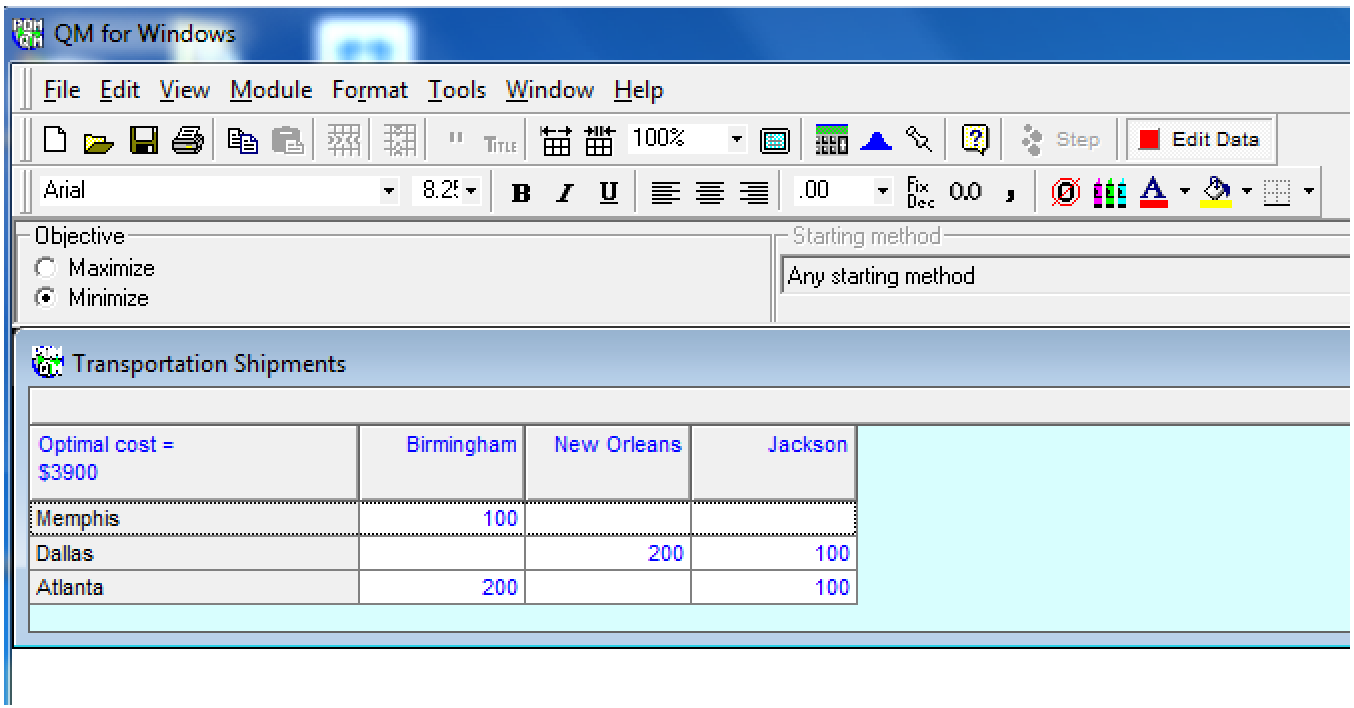

Create data set for Transportation
Title:
Transportation Example
Modify default title
Number of Origins
3
Number of Destinations
3
Objective
Maximize
Minimize
Row names
Column names
Overview
Origin 1, Origin 2, Origin 3,...
a, b, c, d, e, ...
A, B, C, D, E, ...
1, 2, 3, 4, 5, ...
January, February, March, April, ...
Click here to set start month
Other
Cancel
Help
OK

Using QM…



	Birmingham	New Orleans	Jackson	SUPPLY
Memphis	5	4	3	100
Dallas	8	4	3	300
Atlanta	9	7	5	300
DEMAND	300	200	200	

Using QM…

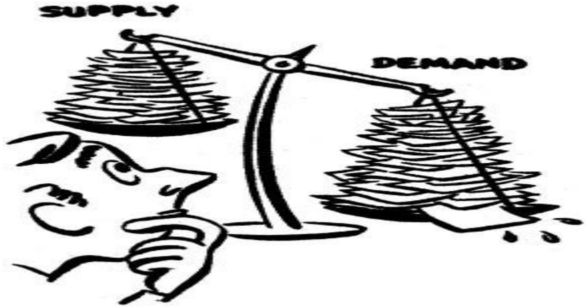

QM for Windows
File Edit View Module Format Tools Window Help
100%
Step
Edit Data
Arial
8.25
.00
Fix Dec
0.0
Objective
Maximize
Minimize
Starting method
Any starting method
Transportation Shipments
Optimal cost = $3900
Birmingham
New Orleans
Jackson
Memphis 100
Dallas 200 100
Atlanta 200 100

Unbalanced Transportation Problems

- In real-life problems, total demand may not be equal to total supply
- These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations*
- If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created
- If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply

Unbalanced Transportation Problems (con't)

- In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped.
- Any units assigned to a dummy destination represent excess capacity
- Any units assigned to a dummy source represent unmet demand



More Than One Optimal Solution

- It is possible for a transportation problem to have multiple optimal solutions
- This means that it is possible to design alternative shipping routes with the same total shipping cost
- In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources

Unacceptable Or Prohibited Routes

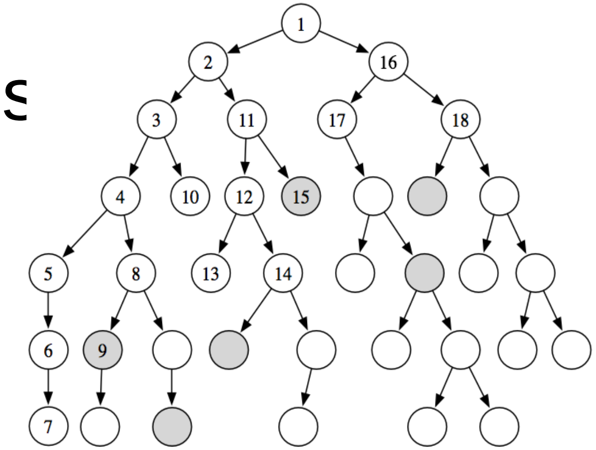
- At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations
  - The problem is said to have an *unacceptable* or *prohibited route*
- In a minimization problem, such a prohibited route is assigned a very high cost to prevent this route from ever being used in the optimal solution
- In a maximization problem, the very high cost used in minimization problems is given a negative sign, turning it into a very bad profit

Facility Location Analysis

- The transportation method is especially useful in helping a firm to decide where to locate a new factory or warehouse
- Each alternative location should be analyzed within the framework of one *overall* distribution system
- The new location that yields the minimum cost for the *entire system* is the one that should be chosen

Special Methods

- Transportation and assignment problems have some special algorithms that can be utilized as well as the general purpose integer linear programming methods including:
  - Branch and bound
  - Modified corner point methods
  - Stepping stone methods
  - Hungarian methods, etc.



Labor Planning

- Labor planning problems address resource needs over time and/or space
- The resources are often restricted to integers such as X number of people
- For example a bank may need a certain number of tellers for time periods of the day to satisfy differing customer demand by time period as shown on the next slide (for a total of 112 needed daily hours)

Labor Planning (con't)

Time Period	# of Tellers Required
9am – 10am	10
10am – 11am	12
11am – Noon	14
Noon – 1pm	16
1pm – 2pm	18
2pm – 3pm	17
3pm – 4pm	15
4pm – 5pm	10

Labor Planning (con't)

- The bank employs up to 12 full time tellers at a cost of $100 per day per teller
- Full time tellers work from 9am until 5pm with 1 hour off for lunch (35 hour work week); half of the full time tellers go to lunch at 11am and the other half at noon
- Part time tellers put in exactly 4 hours per day and can start at any hourly even time slot (between 9am and 1pm); they cost $32 per day (Let P1 be tellers at first time slot, P2 at second time slot, etc.)
- By regulations part time worker hours cannot exceed 50% of the total teller required hours:
  - $4 * (P1 + P2 + P3 + P4 + P5) <= .5 * 112 <= 56$

Labor Planning (con't)

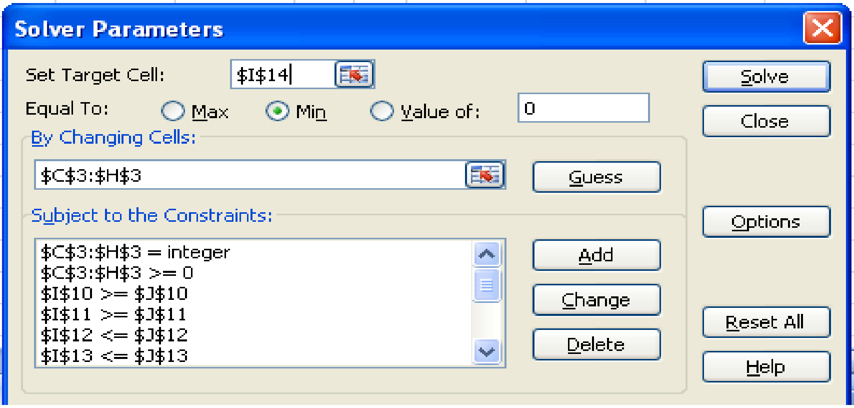
B15

	A	B	C	D	E	F	G	H	I	J	K
1											
2			F	P1	P2	P3	P4	P5			
3		Solution	0	0	0	0	0	0			
4		9-10a	1	1					=$C$3*C4+$D$3*D4+$E$3*E4+$F$3*F4+$G$3*G4+$H$3*H4	10	>=
5		10-11a	1	1	1				=$C$3*C5+$D$3*D5+$E$3*E5+$F$3*F5+$G$3*G5+$H$3*H5	12	>=
6		11a - Noon	0.5	1	1	1			=$C$3*C6+$D$3*D6+$E$3*E6+$F$3*F6+$G$3*G6+$H$3*H6	14	>=
7		Noon - 1p	0.5	1	1	1	1		=$C$3*C7+$D$3*D7+$E$3*E7+$F$3*F7+$G$3*G7+$H$3*H7	16	>=
8		1 - 2p	1		1	1	1	1	=$C$3*C8+$D$3*D8+$E$3*E8+$F$3*F8+$G$3*G8+$H$3*H8	18	>=
9		2-3p	1			1	1	1	=$C$3*C9+$D$3*D9+$E$3*E9+$F$3*F9+$G$3*G9+$H$3*H9	17	>=
10		3-4p	1				1	1	=$C$3*C10+$D$3*D10+$E$3*E10+$F$3*F10+$G$3*G10+$H$3*H10	15	>=
11		4-5p	1					1	=$C$3*C11+$D$3*D11+$E$3*E11+$F$3*F11+$G$3*G11+$H$3*H11	10	>=
12		Full Time	1						=$C$3*C12+$D$3*D12+$E$3*E12+$F$3*F12+$G$3*G12+$H$3*H12	12	<=
13		Part Time		4	4	4	4	4	=$C$3*C13+$D$3*D13+$E$3*E13+$F$3*F13+$G$3*G13+$H$3*H13	56	<=
14		Objective	100	32	32	32	32	32	=$C$3*C14+$D$3*D14+$E$3*E14+$F$3*F14+$G$3*G14+$H$3*H14		Min
15											
16											

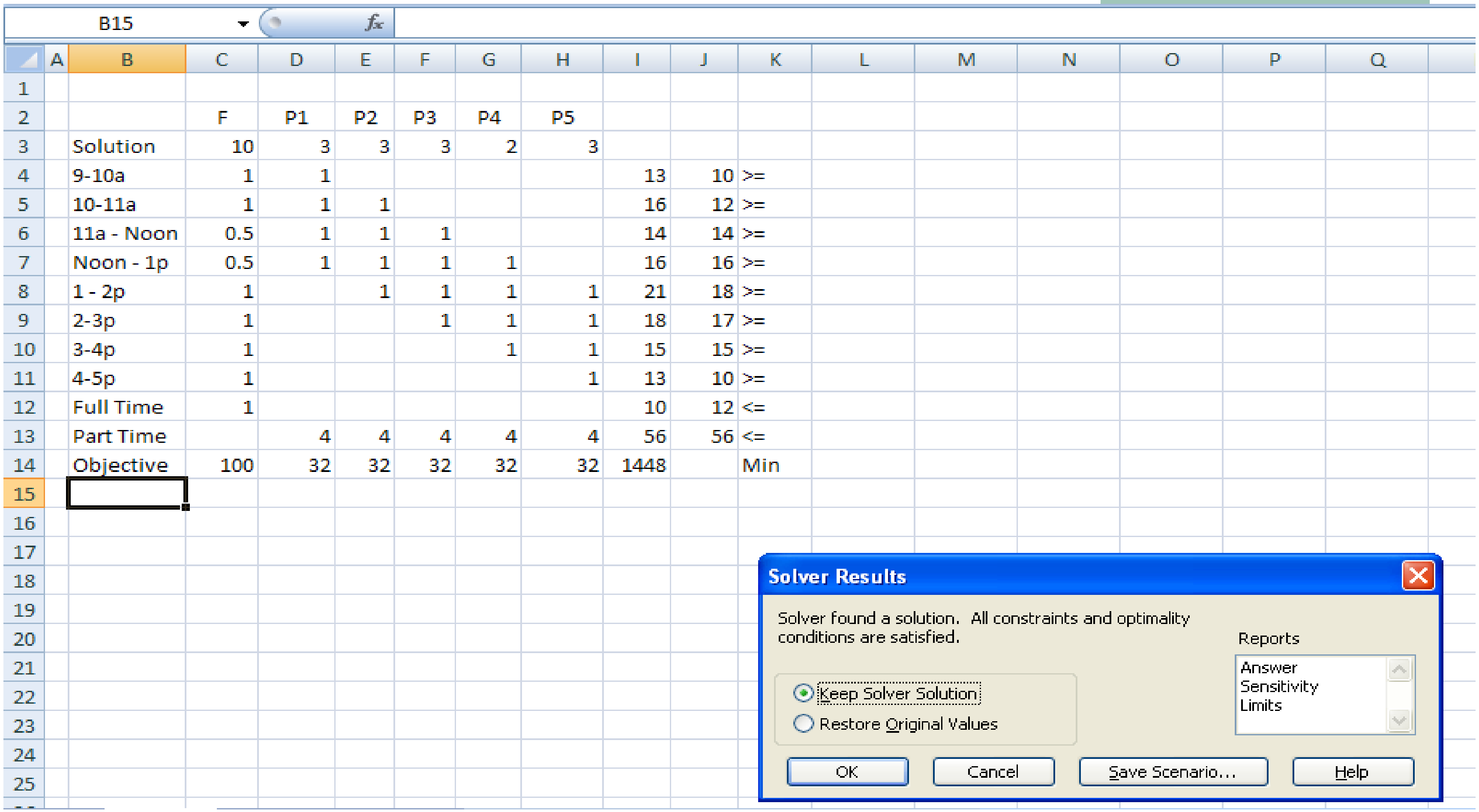
Labor Planning (con't)

I14 =$C$3*C14+$D$3*D14+$E$3*E14+$F$3*F14+$G$3*G14+$H$3*H14

	B	C	D	E	F	G	H	I	J	K
1										
2		F	P1	P2	P3	P4	P5			
3	Solution	0	0	0	0	0	0			
4	9-10a	1	1					0	10	>=
5	10-11a	1	1	1				0	12	>=
6	11a - Noon	0.5	1	1	1			0	14	>=
7	Noon - 1p	0.5	1	1	1	1		0	16	>=
8	1 - 2p	1		1	1	1	1	0	18	>=
9	2-3p	1			1	1	1	0	17	>=
10	3-4p	1				1	1	0	15	>=
11	4-5p	1					1	0	10	>=
12	Full Time	1						0	12	<=
13	Part Time		4	4	4	4	4	0	56	<=
14	Objective	100	32	32	32	32	32	0		Min



Labor Planning (con't)



	F	P1	P2	P3	P4	P5			
Solution	10	3	3	3	2	3			
9-10a	1	1					13	10	>=
10-11a	1	1	1				16	12	>=
11a - Noon	0.5	1	1	1			14	14	>=
Noon - 1p	0.5	1	1	1	1		16	16	>=
1 - 2p	1		1	1	1	1	21	18	>=
2-3p	1			1	1	1	18	17	>=
3-4p	1				1	1	15	15	>=
4-5p	1					1	13	10	>=
Full Time	1						10	12	<=
Part Time		4	4	4	4	4	56	56	<=
Objective	100	32	32	32	32	32	1448		Min

Labor Planning (con’t)

Another solution at $1448 – there are multiple optimal solutions

	A	B	C	D	E	F	G	H	I	J	K
1											
2			F	P1	P2	P3	P4	P5			
3		Solution	10	0	2	7	5	0			
4		9-10a	1	1					10	10	>=
5		10-11a	1	1	1				12	12	>=
6		11a - Noon	0.5	1	1	1			14	14	>=
7		Noon - 1p	0.5	1	1	1	1		19	16	>=
8		1 - 2p	1		1	1	1	1	24	18	>=
9		2-3p	1			1	1	1	22	17	>=
10		3-4p	1				1	1	15	15	>=
11		4-5p	1					1	10	10	>=
12		Full Time	1						10	12	<=
13		Part Time		4	4	4	4	4	56	56	<=
14		Objective	100	32	32	32	32	32	1448		Min
15											
16											

Labor Planning (con't)

Another Optimal solution – which one depends upon starting values for the solution variables, and the order of the constraints !!!

N7

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2			F	P1	P2	P3	P4	P5				
3		Solution	10	6	1	2	5	0				
4		9-10a	1	1					16	10	>=	
5		10-11a	1	1	1				17	12	>=	
6		11a - Noon	0.5	1	1	1			14	14	>=	
7		Noon - 1p	0.5	1	1	1	1		19	16	>=	
8		1 - 2p	1		1	1	1	1	18	18	>=	
9		2-3p	1			1	1	1	17	17	>=	
10		3-4p	1				1	1	15	15	>=	
11		4-5p	1					1	10	10	>=	
12		Full Time	1						10	12	<=	
13		Part Time		4	4	4	4	4	56	56	<=	
14		Objective	100	32	32	32	32	32	1448		Min	
15												

Mixed-Integer Programming Problem Example

- There are many situations in which some of the variables are restricted to be integers and some are not
- As an example consider a chemical company that produces two industrial chemicals
- Xyline must be produced in 50-pound bags
- Hexall is sold by the pound and can be produced in any quantity
- Both xyline and hexall are composed of three ingredients – *A*, *B*, and *C*
- Xyline sells for $85 a bag and hexall for $1.50 per pound

Mixed-Integer Programming Problem Example (con't)

AMOUNT PER 50-POUND BAG OF XYLINE (LB)	AMOUNT PER POUND OF HEXALL (LB)	AMOUNT OF INGREDIENTS AVAILABLE
30	0.5	2,000 lb–ingredient *A*
18	0.4	800 lb–ingredient *B*
2	0.1	200 lb–ingredient *C*

- We want to maximize profit
- We let X = number of 50-pound bags of xyline
- We let Y = number of pounds of hexall
- This is a mixed-integer programming problem as Y is not required to be an integer

Mixed-Integer Programming Problem Example (con't)

- The model is

Maximize profit = $\$85X + \$1.50Y$

subject to

$$30X + 0.5Y \leq 2{,}000$$

$$18X + 0.4Y \leq 800$$

$$2X + 0.1Y \leq 200$$

X, Y >= 0 and X integer

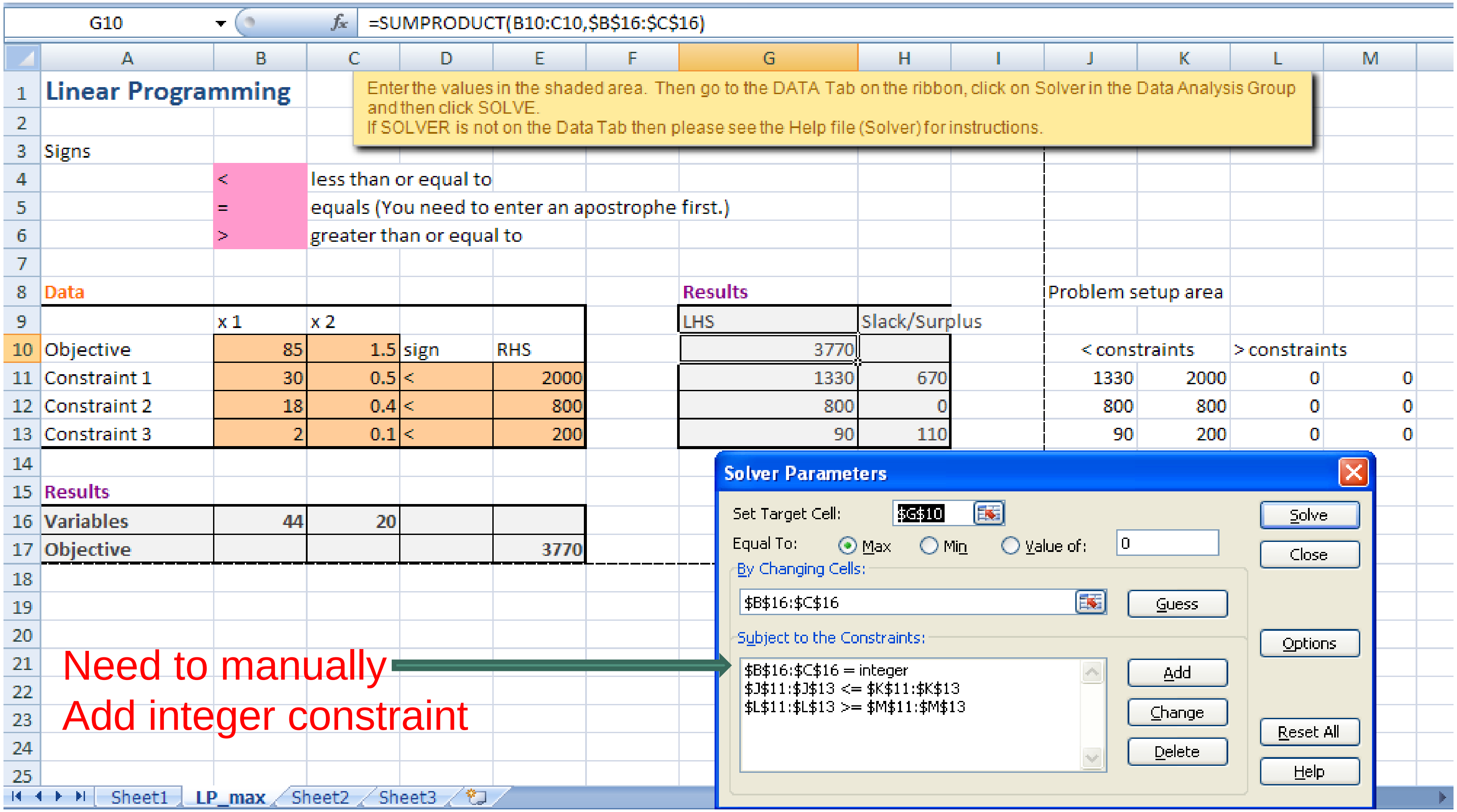
Mixed-Integer Programming Problem Example (con't)

- Using Excel QM

Enter the values in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver) for instructions.

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Mixed Integer example**											
2												
3	**Linear Programming**											
4												
5	Signs											
6		<	less than or equal to									
7		=	equals (You need to enter an apostrophe first.)									
8		>	greater than or equal to									
9												
10	Data						**Results**			Problem setup area		
11		x 1	x 2				LHS	Slack/Surplus				
12	Objective	85	1.5	sign	RHS		0			< constraints		> constraints
13	Constraint 1	30	0.5	<	2000		0	2000		0	2000	0
14	Constraint 2	18	0.4	<	800		0	800		0	800	0
15	Constraint 3	2	0.1	<	200		0	200		0	200	0
16												
17	**Results**											
18	**Variables**											
19	**Objective**				0							
20												

Mixed-Integer Programming Problem Example (con't)



Real World Mixed Integer Problem

[Network Mode Optimization for the DHL Supply Chain, Informs J. on Applied Analytics, Vol 51, No 3 2021]

Objective function

$$\text{Min} \sum_{k \in V} \left\{ f \cdot \Theta_k + \sum_{i \in I_0} \sum_{j \in I_0} x_{ijk} \cdot (c_{ij} + p) - \sum_{i \in I_0} x_{i0k} \cdot p \right\} + \sum_{i \in I} \lambda_i \cdot y_i$$

Subject to:

Degree constraints

$$\sum_{i \in I_0} x_{ijk} = \sum_{h \in I_0} x_{jhk}, \ \forall k \in V, \ \forall j \in I_0$$

$$X_{iik} = 0, \ \forall i \in I_0, \forall k \in V$$

$$\sum_{k \in V} \sum_{j \in I_0} x_{ijk} + y_i = 1, \ \forall i \in I$$

$$\sum_{j \in I_0} x_{0jk} \le 1, \ \forall k \in V$$

Time-window constraints

$$a_j \cdot x_{ijk} \ge s_{ijk} \ge e_j \cdot x_{ijk}, \ \forall i \in I_0, \forall j \in I, \forall k \in V$$

$$s_{ijk} \le \sum_{h \in I_0} s_{hik} + x_{ijk} \cdot \left(\frac{d_{ij}}{g} + \mu_i \right) + b \cdot l_{jk}, \forall i \in I, \forall j \in I_0, \forall k \in V$$

$$s_{ijk} \ge \sum_{h \in I_0} s_{hik} + x_{ijk} \cdot \left(\frac{d_{ij}}{g} + \mu_i \right) + b \cdot l_{jk} - (1 - x_{ijk}) \cdot M, \forall i \in I, \forall j \in I_0, \forall k \in V$$

Layover constraints

$$\sum_{i \in I_0} \sum_{j \in I_0} x_{ijk} \cdot \left(\frac{d_{ij}}{g} + \mu_i \right) \le \delta + 2 \cdot \delta \cdot z_k, \ \forall k \in V$$

$$\sum_{i \in I_0} \sum_{j \in I_0} x_{ijk} \cdot \left(\frac{d_{ij}}{g} + \mu_i \right) \le 2 \cdot \delta + \delta \cdot r_k, \ \forall k \in V$$

$$r_k + z_k = \sum_{j \in I_0} l_{jk}, \forall k \in V$$

$$l_{jk} \le \sum_{i \in I_0} x_{ijk}, \forall j \in I_0, \forall k \in V$$

Truck-capacity constraints

$$q + (w_i - q) \cdot \sum_{k \in V} x_{0ik} \ge u_i \ge w_i, \ \forall i \in I$$

$$u_i - u_j + q \cdot \left(\sum_{k \in V} x_{ijk} \right) \le q - w_j, \forall i, j \in I$$

Maximum-travel-time constraints

$$t^{max} \ge t_k \ge \sum_{i \in I_0} s_{i0k} - \sum_{j \in I_0} s_{0jk} + \sum_{j \in I_0} x_{0jk} \cdot \left(\frac{d_{0j}}{g} \right), \forall k \in V$$

$$\Theta_k \ge \frac{t_k}{24}, \forall k \in V$$

Intranode-distance constraints

$$d_{ij} \cdot x_{ijk} \le d^{max}, \forall i, j \in I_0, \forall k \in V$$

Symmetry breaking inequalities

$$t_k \ge t_{k+1}, \forall k \in V$$

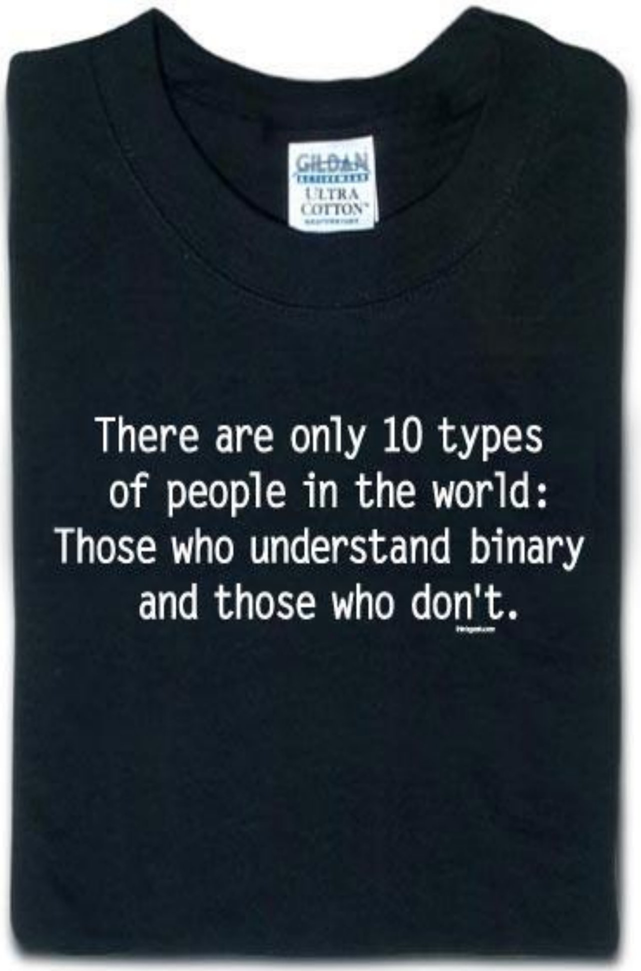
Integrality and nonnegativity constraints

$$x_{ijk}, y_i, l_{ik}, r_k, z_k \in \{0,1\}, \ y_0 = 0, \forall i \in I_0, \forall j \in I_0, \forall k \in V$$

$$t_k \ge 0, \ \Theta_k \in Z_+, \ s_{ijk} \ge 0, \ u_i \ge 0, \ , \forall i \in I, \forall j \in I_0, \forall k \in V$$

Modeling With 0-1 (Binary) Variables

- Let's demonstrate how 0-1 variables can be used to model several diverse situations
- Typically a 0-1 variable is assigned a value of 0 if a certain condition is not met and a 1 if the condition is met
- This is also called a *binary variable*



The Assignment Problem

- Another common LP algorithm is the assignment method
- Each assignment problem has associated with it a table, or matrix
- Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things to which we want them assigned
- The numbers in the table are the costs associated with each particular assignment
- An assignment problem can be viewed as a transportation problem in which the capacity from each source is 1 and the demand at each destination is 1

Assignment Model Approach

- The Knucklehead's Fix-It Shop has three rush projects to repair
- The shop has three repair persons with different talents and abilities (Larry, Curley, and Moe)
- The owner has estimates of wage costs for each worker for each project
- The owner's objective is to assign the three projects to the workers in a way that will result in the lowest cost to the shop
- Each project will be assigned exclusively to one worker

Assignment Problem Approach (con't)

Estimated Project Repair Costs for the Fix-It Shop Assignment Problem

	PROJECT		
PERSON	1	2	3
Larry	$11	$14	$6
Curley	8	10	11
Moe	9	12	7

PERSON	PROJECT 1	2	3
Larry	$11	$14	$6
Curley	8	10	11
Moe	9	12	7

- X_{ij} = 1 if person i is assigned to job j, else 0

- Minimize $Z = 11X_{11} + 14X_{12} + 6X_{13} +$
- $8X_{21} + 10X_{22} + 11X_{23} +$
- $9X_{31} + 12X_{32} + 7X_{33}$
- Subject to:
  - $X_{i1} + X_{i2} + X_{i3} <= 1$ for I = 1 to 3
  - X_{ij} >=0 and X_{ij} <=1 and X_{ij} is an integer (X_{ij} are binary)

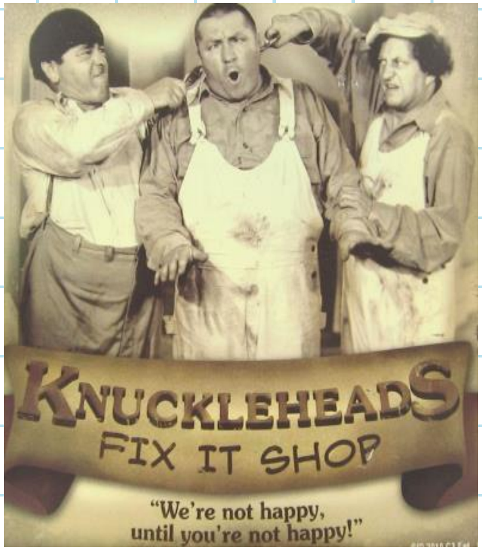
Assignment Model Approach (con't)

	B	C	D	E	F
	Cost for Assignments				
	Project 1	Project 2	Project 3		
	11	14	6		
	8	10	11		
	9	12	7		
		Made			
	Project 1	Project 2	Project 3	Total projects	Supply
	0	0	1	1	1
	0	1	0	1	1
	1	0	0	1	1
	1	1	1		
	1	1	1		
	25				

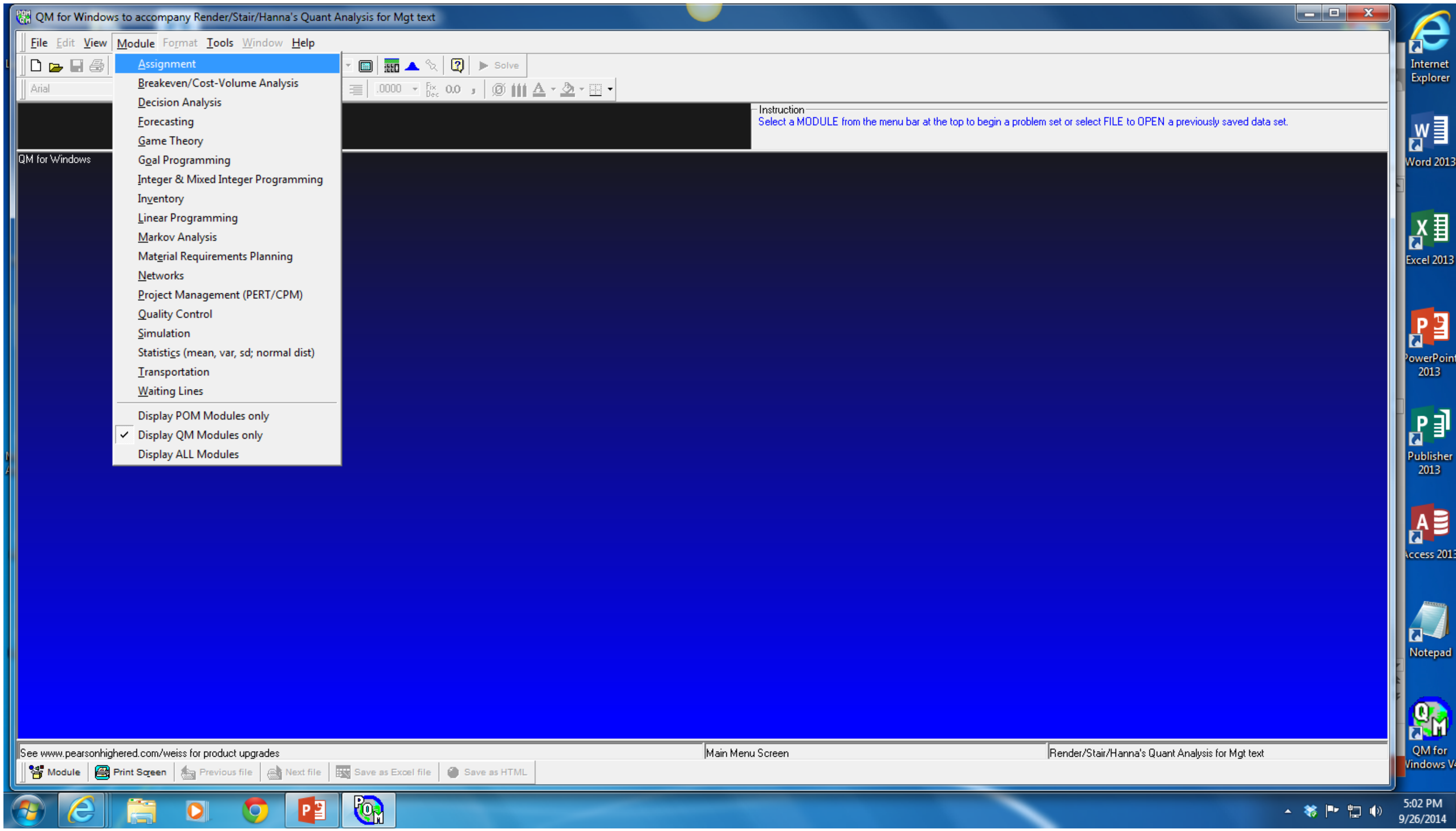
	E
10	=SUM(B10:D10)

	B
13	=SUM(B10:B12)

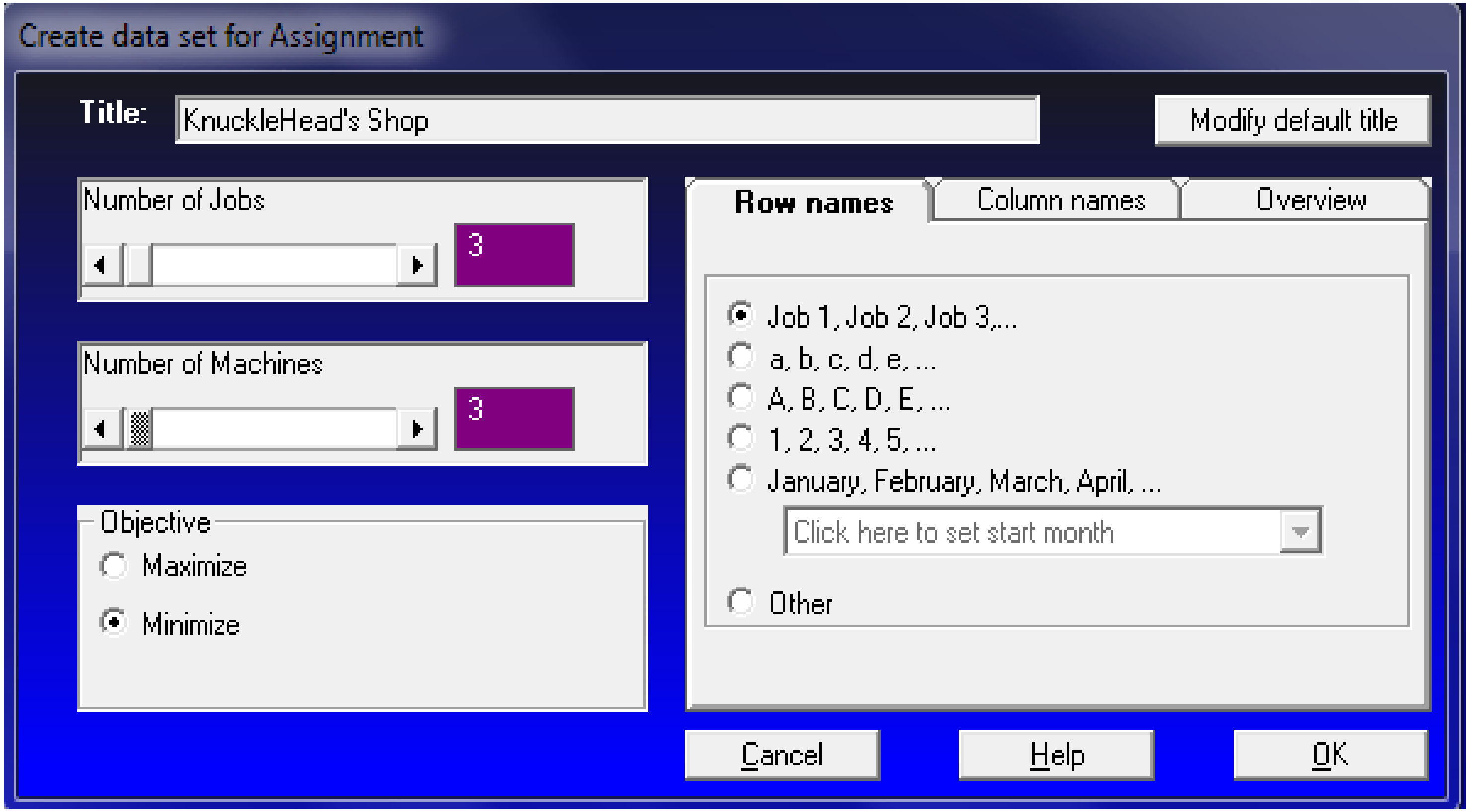
	B
16	=SUMPRODUCT(B3:D5,B10:D12)



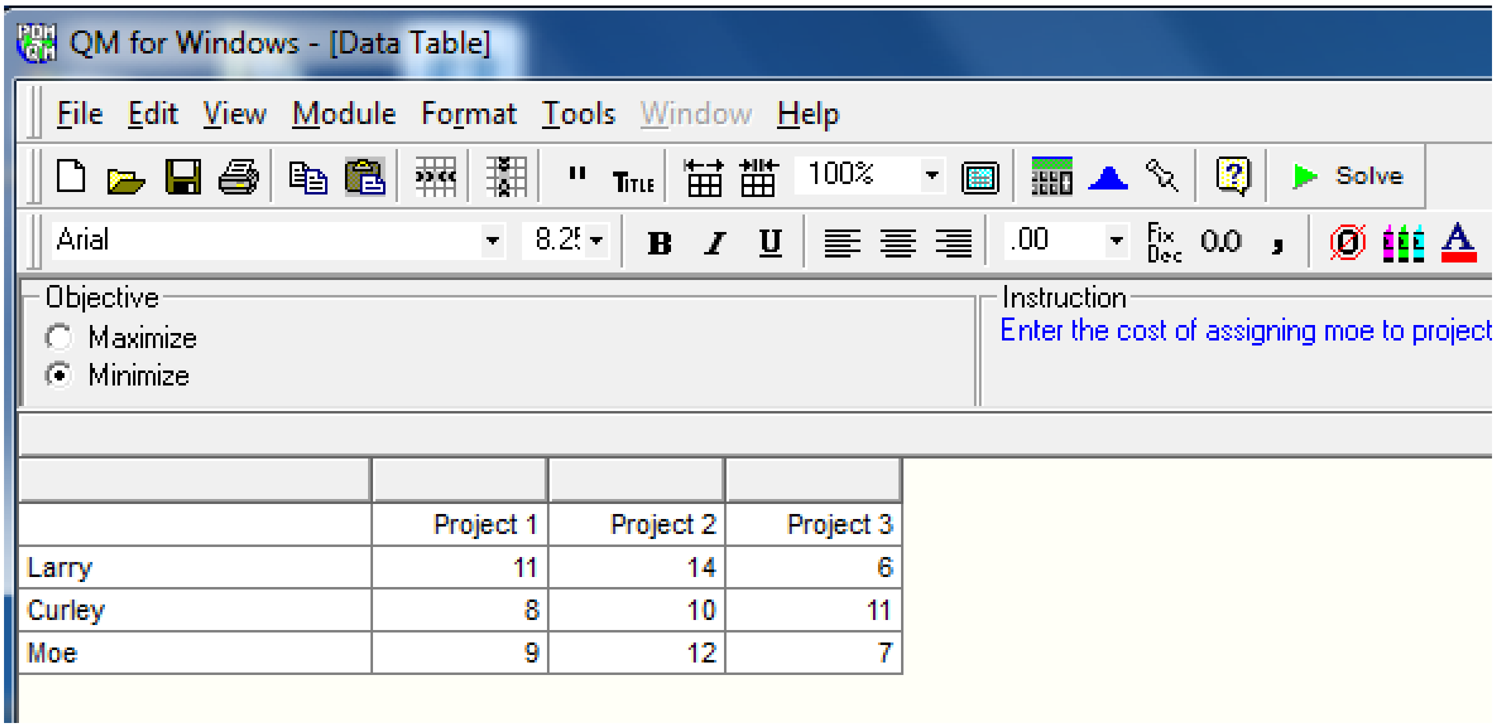
Using QM…


QM for Windows to accompany Render/Stair/Hanna's Quant Analysis for Mgt text
File Edit View Module Format Tools Window Help
Assignment
Breakeven/Cost-Volume Analysis
Decision Analysis
Forecasting
Game Theory
Goal Programming
Integer & Mixed Integer Programming
Inventory
Linear Programming
Markov Analysis
Material Requirements Planning
Networks
Project Management (PERT/CPM)
Quality Control
Simulation
Statistics (mean, var, sd; normal dist)
Transportation
Waiting Lines
Display POM Modules only
Display QM Modules only
Display ALL Modules
Solve
Instruction
Select a MODULE from the menu bar at the top to begin a problem set or select FILE to OPEN a previously saved data set.
QM for Windows
See www.pearsonhighered.com/weiss for product upgrades
Main Menu Screen
Render/Stair/Hanna's Quant Analysis for Mgt text
Module
Print Screen
Previous file
Next file
Save as Excel file
Save as HTML
Internet Explorer
Word 2013
Excel 2013
PowerPoint 2013
Publisher 2013
Access 2013
Notepad
QM for Windows V4
5:02 PM
9/26/2014

Using QM…

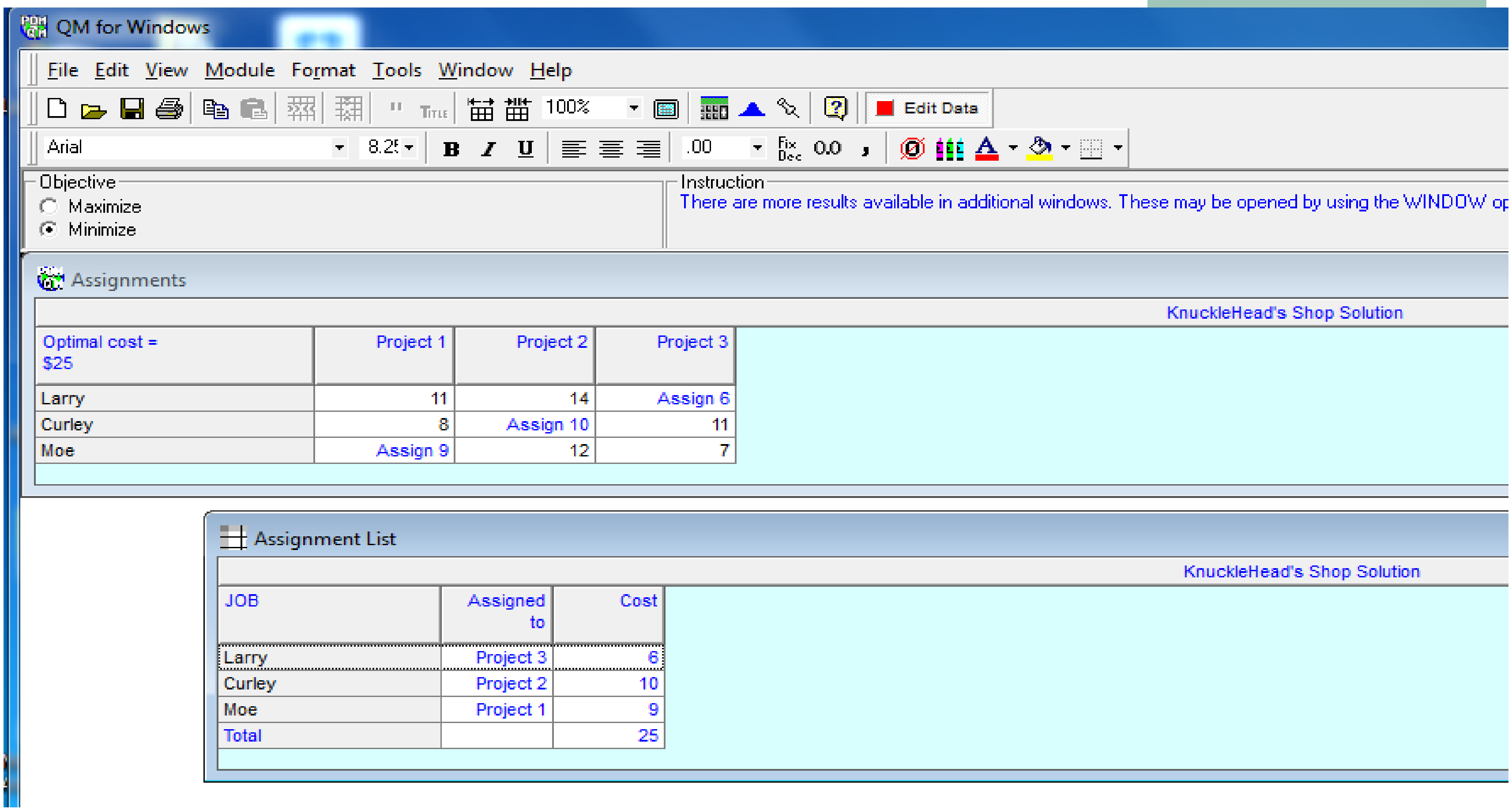

Create data set for Assignment
Title:
KnuckleHead's Shop
Modify default title
Number of Jobs
3
Number of Machines
3
Objective
Maximize
Minimize
Row names
Column names
Overview
Job 1, Job 2, Job 3,...
a, b, c, d, e, ...
A, B, C, D, E, ...
1, 2, 3, 4, 5, ...
January, February, March, April, ...
Click here to set start month
Other
Cancel
Help
OK

Using QM…



	Project 1	Project 2	Project 3
Larry	11	14	6
Curley	8	10	11
Moe	9	12	7

Using QM…


QM for Windows
File Edit View Module Format Tools Window Help
100%
Edit Data
Arial
8.25
.00
Fix Dec
0.0
Objective
Maximize
Minimize
Instruction
There are more results available in additional windows. These may be opened by using the WINDOW op
Assignments
KnuckleHead's Shop Solution
Optimal cost = $25
Project 1
Project 2
Project 3
Larry 11 14 Assign 6
Curley 8 Assign 10 11
Moe Assign 9 12 7
Assignment List
KnuckleHead's Shop Solution
JOB
Assigned to
Cost
Larry Project 3 6
Curley Project 2 10
Moe Project 1 9
Total 25

Fixed-Charge Problem Example

- Often businesses are faced with decisions involving a fixed charge that will affect the cost of future operations
- A manufacturing company is planning to build at least one new plant and three cities are being considered:
  - Baytown, Texas
  - Lake Charles, Louisiana
  - Mobile, Alabama
- Once the plant or plants are built, the company wants to have capacity to produce at least 38,000 units each year

Fixed-Charge Problem Example (con't)

- Fixed and variable costs for Manufacturing Co.

SITE	ANNUAL FIXED COST	VARIABLE COST PER UNIT	ANNUAL CAPACITY
Baytown, TX	$340,000	$32	21,000
Lake Charles, LA	$270,000	$33	20,000
Mobile, AL	$290,000	$30	19,000

SITE	ANNUAL FIXED COST	VARIABLE COST PER UNIT	ANNUAL CAPACITY
Baytown, TX	$340,000	$32	21,000
Lake Charles, LA	$270,000	$33	20,000
Mobile, AL	$290,000	$30	19,000

- Management decisions ? [need 38,000/yr]
- What are the variables ?
- Use of binary variables?

- Do not look ahead !

Fixed-Charge Problem Example (con't)

- We can define the decision variables as

$$X_1 = \begin{cases} 1 \text{ if factory is built in Baytown} \\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ factory is built in Lake Charles} \\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if factory is built in Mobile} \\ 0 \text{ otherwise} \end{cases}$$

X_4 = number of units produced at Baytown plant

X_5 = number of units produced at Lake Charles plant

X_6 = number of units produced at Mobile plant

Fixed-Charge Problem Example (con't)

- The integer programming formulation becomes

Minimize cost = $340{,}000X_1 + 270{,}000X_2 + 290{,}000X_3 + 32X_4 + 33X_5 + 30X_6$

subject to

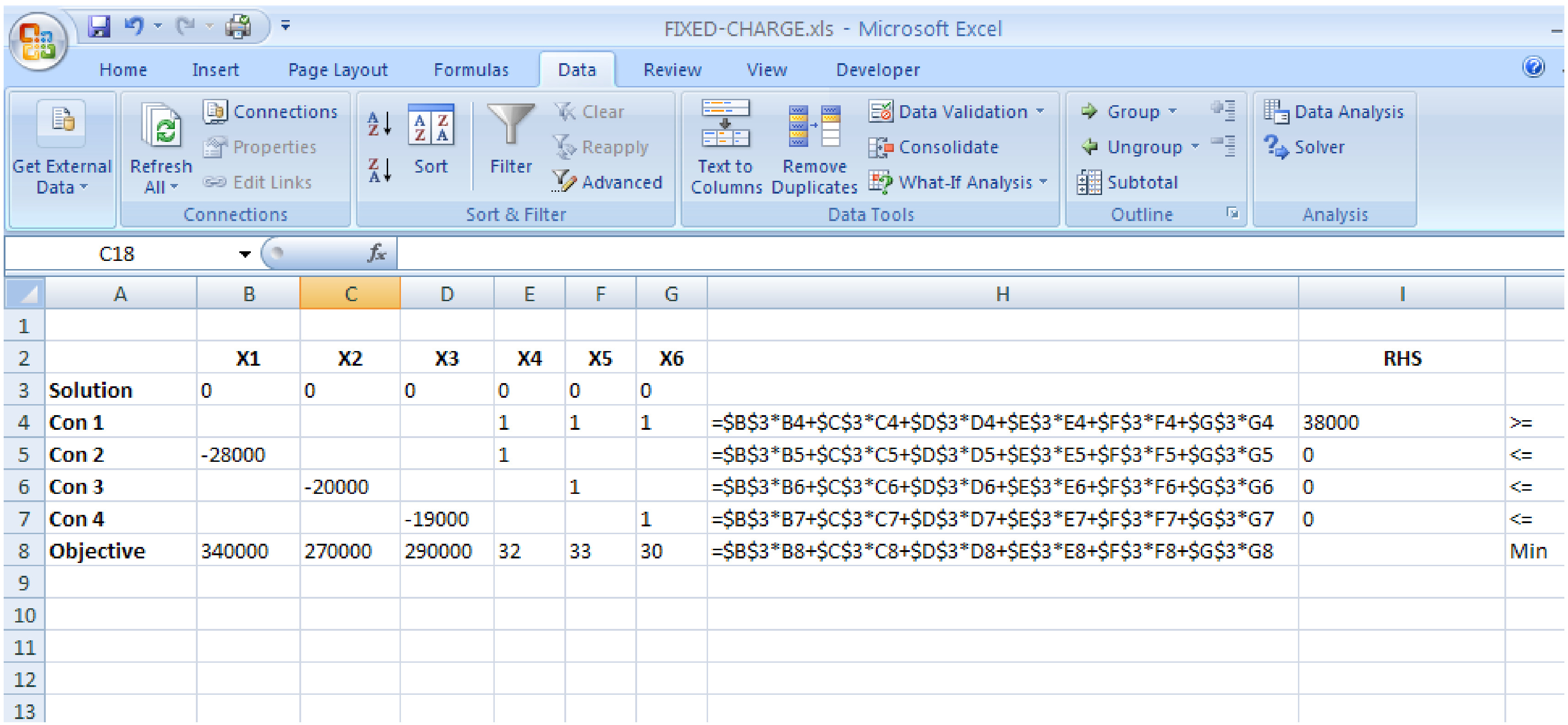
$$
\begin{aligned}
X_4 + X_5 + X_6 &\geq 38{,}000 \\
X_4 &\leq 21{,}000X_1 \\
X_5 &\leq 20{,}000X_2 \\
X_6 &\leq 19{,}000X_3 \\
X_1, X_2, X_3 &= 0 \text{ or } 1; \\
X_4, X_5, X_6 &\geq 0 \text{ and integer}
\end{aligned}
$$

Note capacity constraints expressed in terms of binary variables

- The optimal solution is

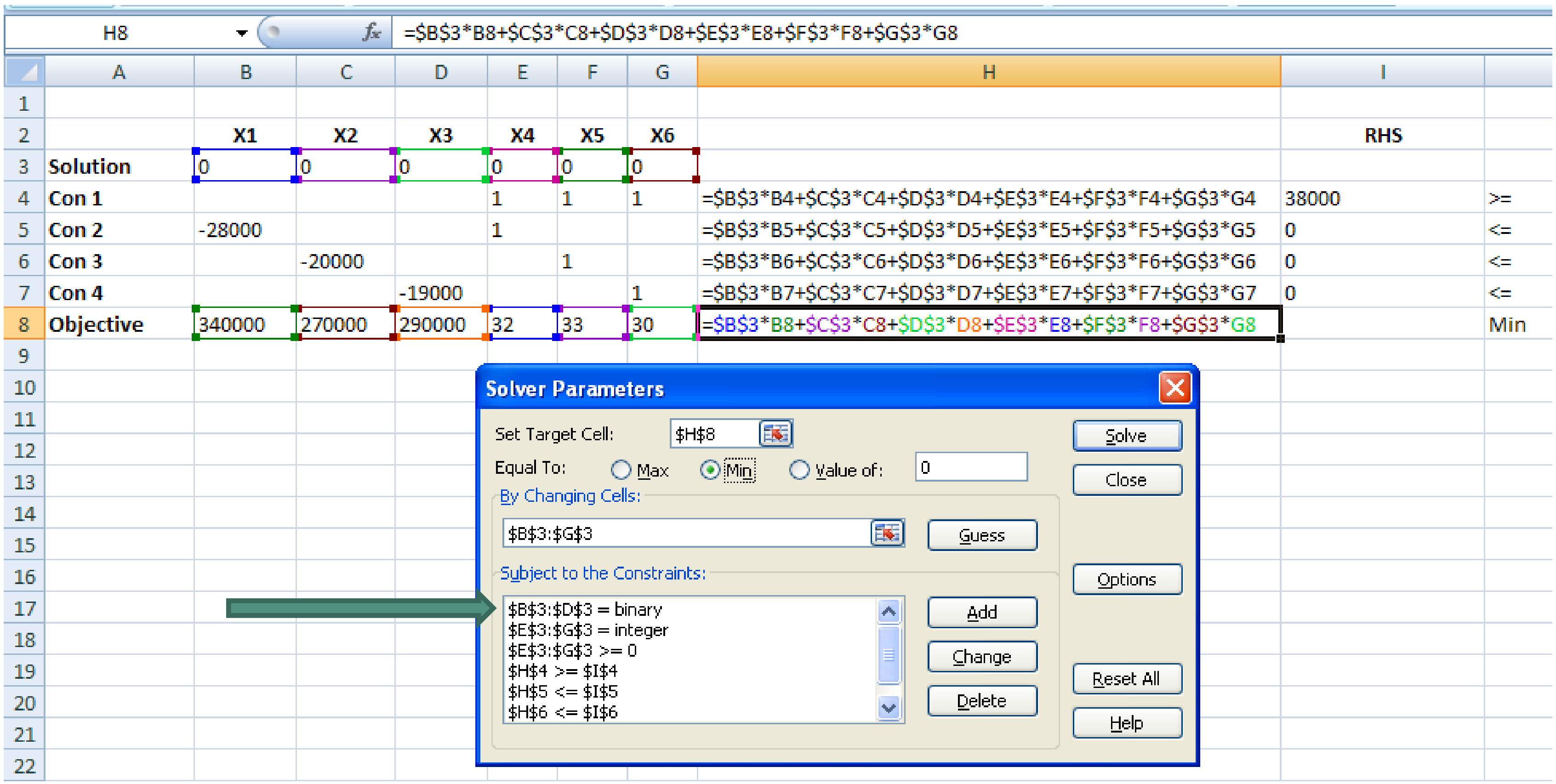
$X_1 = 0$, $X_2 = 1$, $X_3 = 1$ [build Lake Charles & Mobile Plants]
$X_4 = 0$, $X_5 = 19{,}000$, $X_6 = 19{,}000$
Objective function value = $1,757,000

Fixed-Charge Problem Example (con't)

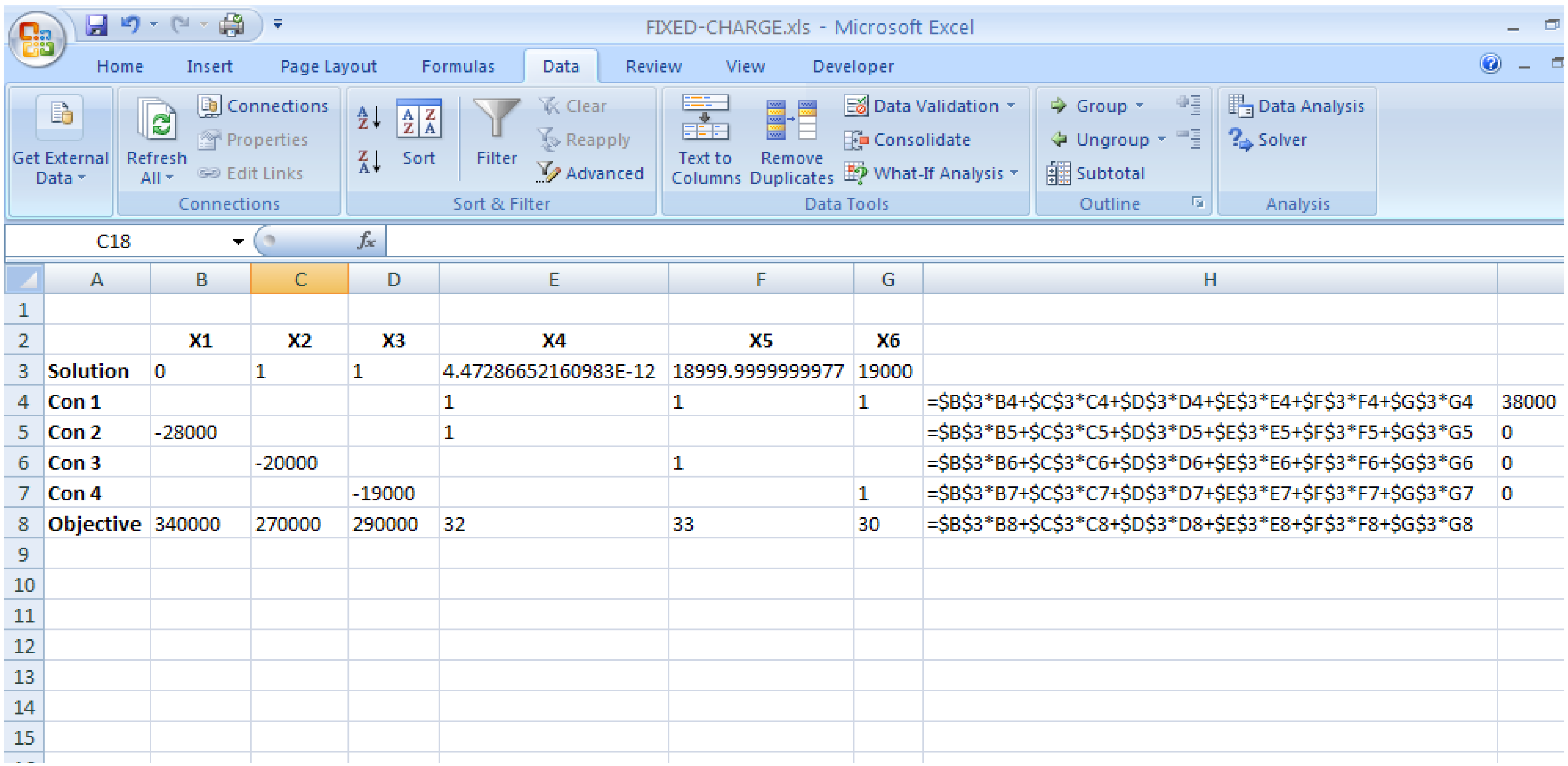


	A	B	C	D	E	F	G	H	I	
1										
2		**X1**	**X2**	**X3**	**X4**	**X5**	**X6**		**RHS**	
3	**Solution**	0	0	0	0	0	0			
4	**Con 1**				1	1	1	=$B$3*B4+$C$3*C4+$D$3*D4+$E$3*E4+$F$3*F4+$G$3*G4	38000	>=
5	**Con 2**	-28000			1			=$B$3*B5+$C$3*C5+$D$3*D5+$E$3*E5+$F$3*F5+$G$3*G5	0	<=
6	**Con 3**		-20000			1		=$B$3*B6+$C$3*C6+$D$3*D6+$E$3*E6+$F$3*F6+$G$3*G6	0	<=
7	**Con 4**			-19000			1	=$B$3*B7+$C$3*C7+$D$3*D7+$E$3*E7+$F$3*F7+$G$3*G7	0	<=
8	**Objective**	340000	270000	290000	32	33	30	=$B$3*B8+$C$3*C8+$D$3*D8+$E$3*E8+$F$3*F8+$G$3*G8		Min
9										
10										
11										
12										
13										

Fixed-Charge Problem Example (con't)



Fixed-Charge Problem Example (con't)



	A	B	C	D	E	F	G	H	
1									
2		X1	X2	X3	X4	X5	X6		
3	Solution	0	1	1	4.47286652160983E-12	18999.9999999977	19000		
4	Con 1				1	1	1	=$B$3*B4+$C$3*C4+$D$3*D4+$E$3*E4+$F$3*F4+$G$3*G4	38000
5	Con 2	-28000			1			=$B$3*B5+$C$3*C5+$D$3*D5+$E$3*E5+$F$3*F5+$G$3*G5	0
6	Con 3		-20000			1		=$B$3*B6+$C$3*C6+$D$3*D6+$E$3*E6+$F$3*F6+$G$3*G6	0
7	Con 4			-19000			1	=$B$3*B7+$C$3*C7+$D$3*D7+$E$3*E7+$F$3*F7+$G$3*G7	0
8	Objective	340000	270000	290000	32	33	30	=$B$3*B8+$C$3*C8+$D$3*D8+$E$3*E8+$F$3*F8+$G$3*G8	

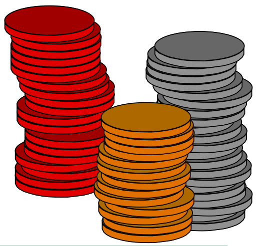
Portfolio Optimization

- Portfolio problems can be regular LP problems or a type of integer problems
  - Maximize the return from a number of financial investments subject to risk and diversity constraints – regular LP
  - Maximize the return on projects by selecting which projects to work on within risk and budget constraints – a zero/one type of integer programming problem (binary programming)

Net Present Value

- $NPV = \sum (B - C)_t/(1+i)^t$
- Where $(B\text{-}C)_t$ is the benefit minus the cost for period t
- i is the interest rate (cost of borrowing money or opportunity cost for other uses of cash)
- For NPV, benefit minus cost is more formally revenue (cash in) minus expenditures (cash out)

NPV Example

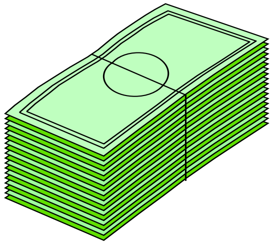


Year	Benefit	Cost	B-C	Discounted B-C
1	$0.00	$175,000.00	-$175,000.00	-$159,090.91
2	$0.00	$175,000.00	-$175,000.00	-$144,628.10
3	$50,000.00	$25,000.00	$25,000.00	$18,782.87
4	$100,000.00	$10,000.00	$90,000.00	$61,471.21
5	$100,000.00	$10,000.00	$90,000.00	$55,882.92
6	$100,000.00	$10,000.00	$90,000.00	$50,802.65
7	$100,000.00	$10,000.00	$90,000.00	$46,184.23
8	$100,000.00	$10,000.00	$90,000.00	$41,985.66
9	$100,000.00	$10,000.00	$90,000.00	$38,168.79
10	$100,000.00	$10,000.00	$90,000.00	$34,698.90
Total	$750,000.00	$445,000.00	$305,000.00	$44,258.22
		Interest =	0.1	

Internal Rate of Return (IRR)

- Another similar project financial evaluation technique is called the ***internal rate of return*** (IRR)
- This metric is better than NPV since it is not as sensitive to the uncertainties of future benefits and costs and to the future interest rates
- The internal rate of return is the value of the interest rate that yields a zero value for NPV
  - This can be calculated in spreadsheet programs by using built-in “solver” tools. Since in reality a quadratic equation is being solved, multiple IRR values could be found. Thus one must impose additional constraints on the solution (such as IRR is positive, or in a given range).

IRR Example



Year	Benefit	Cost	B-C	Discounted B-C
1	$0.00	$175,000.00	-$175,000.00	-$154,728.74
2	$0.00	$175,000.00	-$175,000.00	-$136,805.61
3	$50,000.00	$25,000.00	$25,000.00	$17,279.80
4	$100,000.00	$10,000.00	$90,000.00	$55,001.46
5	$100,000.00	$10,000.00	$90,000.00	$48,630.33
6	$100,000.00	$10,000.00	$90,000.00	$42,997.19
7	$100,000.00	$10,000.00	$90,000.00	$38,016.58
8	$100,000.00	$10,000.00	$90,000.00	$33,612.90
9	$100,000.00	$10,000.00	$90,000.00	$29,719.32
10	$100,000.00	$10,000.00	$90,000.00	$26,276.76
Total	$750,000.00	$445,000.00	$305,000.00	$0.00
		Interest =	0.131011619	

Projects with the same net present value may have different internal rates of return

Case 1

Period	Benefit	Cost	B-C	Discounted B-C
1	0	70	-70	-$60.87
2	0	50	-50	-$37.81
3	20	30	-10	-$6.58
4	90	0	90	$51.46
5	120	0	120	$59.66
			NPV:	$5.87
		Interest:	0.15	
		IRR:	0.17	

Case 2

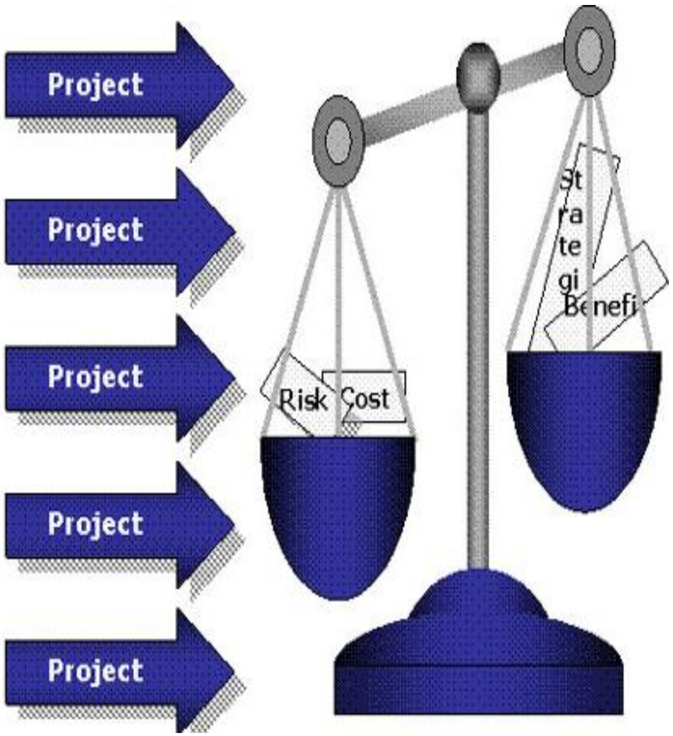
Period	Benefit	Cost	B-C	Discounted B-C
1	0	20	-20	-$17.39
2	0	40	-40	-$30.25
3	20	50	-30	-$19.73
4	90	55	35	$20.01
5	120	12.95	107.05	$53.22
			NPV:	$5.87
		Interest:	0.15	
		IRR:	0.19	

Example Project Portfolio Data

Project	IRR	Risk Factor	Cost ($)
1	0.35	0.15	200
2	0.25	0.05	500
3	0.3	0.5	700
4	0.15	0.3	300
5	0.28	0.25	400
6	0.25	0.3	900
7	0.2	0.2	600
8	0.3	0.1	800

Constraints

- Must balance risk and reward
- Cannot exceed budget of $3000
- Risk must be within limit of “management reserve” (percentage of overall budget, 20% in this example - $600)

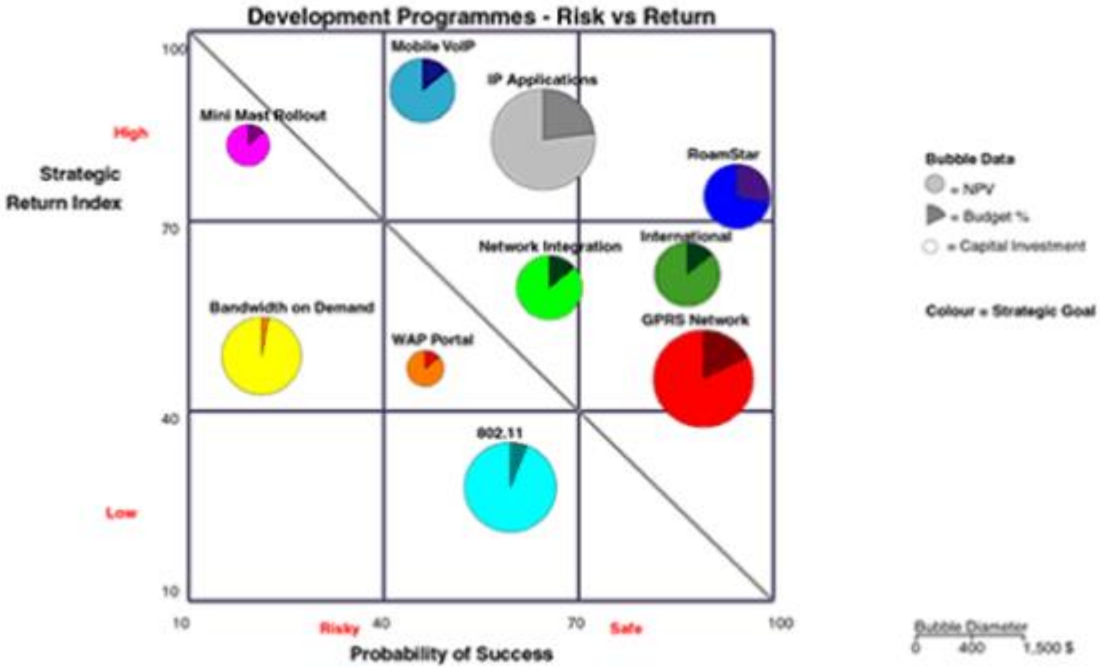


Which projects would you do ?

Budget less than $3000 Risk Contingency <= $600

Project	IRR	Risk Factor	Cost ($)
1	0.35	0.15	200
2	0.25	0.05	500
3	0.3	0.5	700
4	0.15	0.3	300
5	0.28	0.25	400
6	0.25	0.3	900
7	0.2	0.2	600
8	0.3	0.1	800

- What are the variables ?
- Use of binary variables?



- Do not look ahead !

Calculating Return and Risk

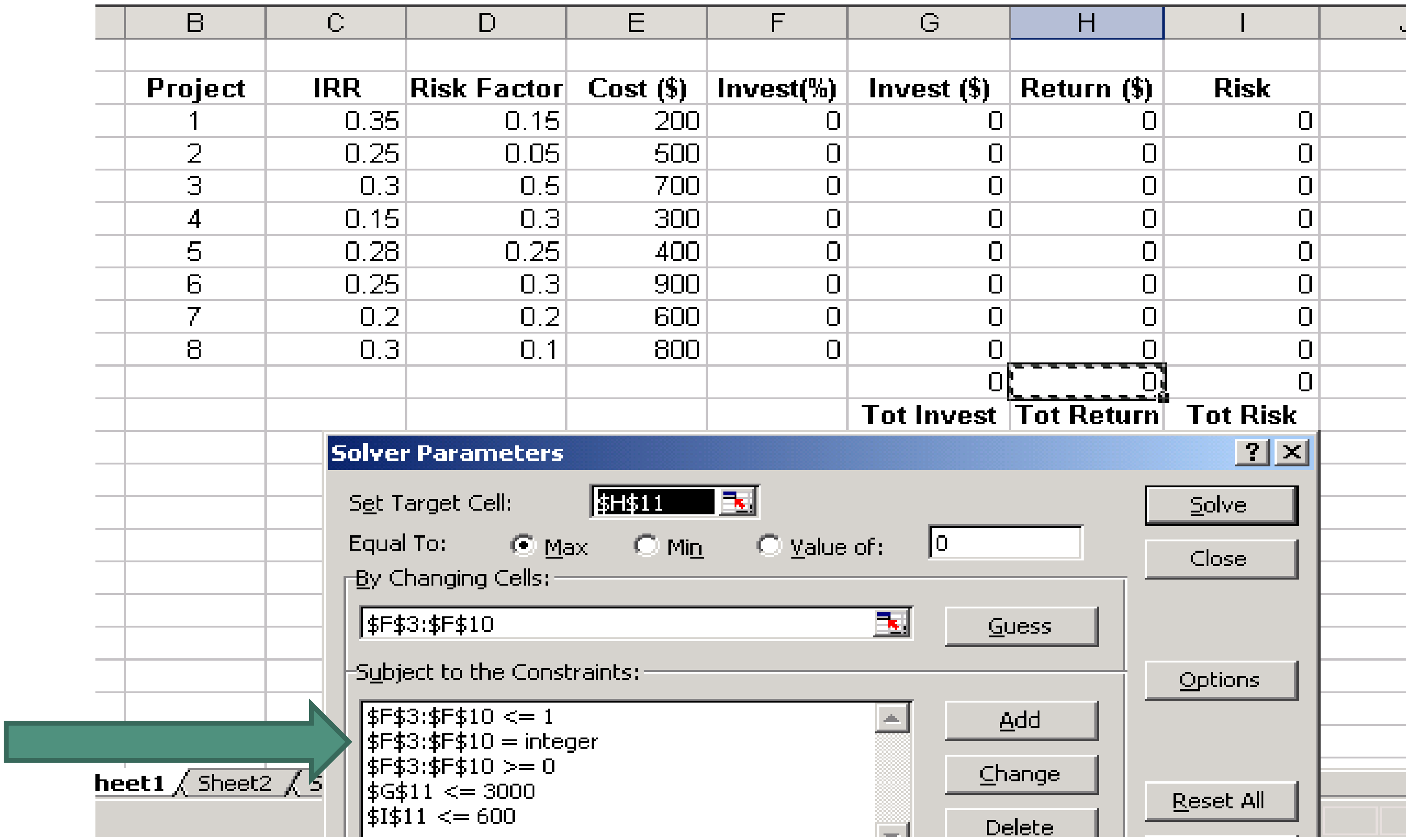
[column E is a binary value]

Microsoft Excel - portfolio_opt.xls

H28

	A	B	C	D	E	F	G	H
1								
2	**Project**	**IRR**	**Risk Factor**	**Cost ($)**	**Invest(%)**	**Invest ($)**	**Return ($)**	**Risk**
3	1	0.35	0.15	200	0	=E3*D3	=E3*B3*D3	=E3*C3*D3
4	2	0.25	0.05	500	0	=E4*D4	=E4*B4*D4	=E4*C4*D4
5	3	0.3	0.5	700	0	=E5*D5	=E5*B5*D5	=E5*C5*D5
6	4	0.15	0.3	300	0	=E6*D6	=E6*B6*D6	=E6*C6*D6
7	5	0.28	0.25	400	0	=E7*D7	=E7*B7*D7	=E7*C7*D7
8	6	0.25	0.3	900	0	=E8*D8	=E8*B8*D8	=E8*C8*D8
9	7	0.2	0.2	600	0	=E9*D9	=E9*B9*D9	=E9*C9*D9
10	8	0.3	0.1	800	0	=E10*D10	=E10*B10*D10	=E10*C10*D10
11						=SUM(F3:F10)	=SUM(G3:G10)	=SUM(H3:H10)
12						**Tot Invest**	**Tot Return**	**Tot Risk**
13								
14								
15								
16								

Using Excel Solver to Set Up Constraints and Get Solution [newer versions of Excel have “bin” constraint]



Project	IRR	Risk Factor	Cost ($)	Invest(%)	Invest ($)	Return ($)	Risk
1	0.35	0.15	200	0	0	0	0
2	0.25	0.05	500	0	0	0	0
3	0.3	0.5	700	0	0	0	0
4	0.15	0.3	300	0	0	0	0
5	0.28	0.25	400	0	0	0	0
6	0.25	0.3	900	0	0	0	0
7	0.2	0.2	600	0	0	0	0
8	0.3	0.1	800	0	0	0	0
					0	0	0
					Tot Invest	Tot Return	Tot Risk

Answers
[not doing projects 3,4,5]

	B	C	D	E	F	G	H	I
	Project	IRR	Risk Factor	Cost ($)	Invest(%)	Invest ($)	Return ($)	Risk
	1	0.35	0.15	200	1	200	70	30
	2	0.25	0.05	500	1	500	125	25
	3	0.3	0.5	700	0	0	0	0
	4	0.15	0.3	300	0	0	0	0
	5	0.28	0.25	400	0	0	0	0
	6	0.25	0.3	900	1	900	225	270
	7	0.2	0.2	600	1	600	120	120
	8	0.3	0.1	800	1	800	240	80
						3000	780	525
						Tot Invest	Tot Return	Tot Risk

Limits on Alternatives Selected

- Suppose it is required to select no more than Z of the projects *regardless* of the funds available
- This would require adding a constraint

$$X_1 + X_2 + X_3 + \ldots \leq Z$$

- If they had to fund *exactly* Z of the projects the constraint would be

$$X_1 + X_2 + X_3 + \ldots = Z$$

Dependent Selections

- At times the selection of one project depends on the selection of another project
- Suppose project 1 could only be done if the project 2 was also done
- The following constrain would force this to occur

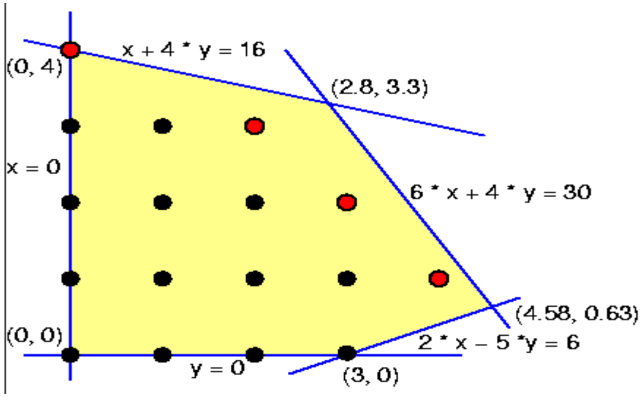
$$X_1 \leq X_2 \quad \text{or} \quad X_1 - X_2 \leq 0$$

- If we wished for project 1 and project 2 to either both be selected or both not be selected, the constraint would be

$$X_1 = X_2 \quad \text{or} \quad X_1 - X_2 = 0$$

Homework

- Textbook Chapter 9 thru section 9.4
- Discussion questions from chapter 9: 2, 3, 4
- Project Seven →



Project 7

- A aero company uses three plants to manufacture amphibious airplane floats
- Let x1 be the batches of big float batches per week and let x2 be the number of small float batches per week
- The big floats contribute 3 units to profit per batch and the small floats contribute 2 units to profit per batch

Project 7 (con't)

- Plant 1 can produce up to 4 batches of x1 per week; plant 2 can produce up to 6 units of x2 per week; plant 3 can produce up to 18 batches per week in the ratio of 3 of x1 and 2 of x2
- What is the optimal mix of big float and small float batches to make each week?
- The number of float batches must be integers
- Show your setup and solution in Excel or QM