Management Science

Decision Analysis

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Session Objectives

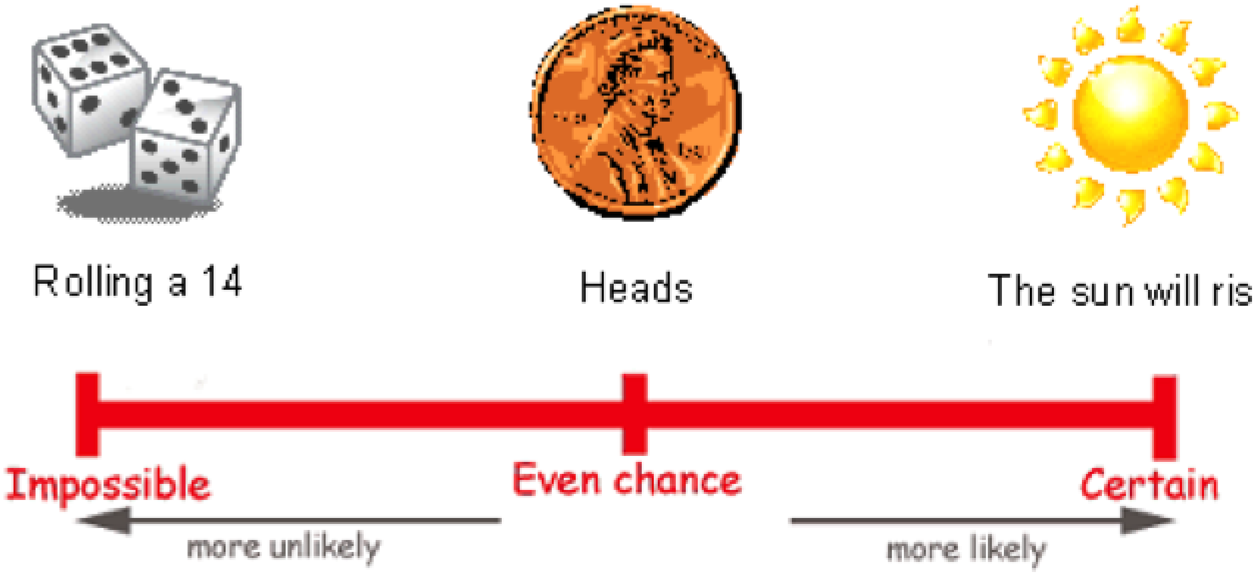
- List the steps of the decision-making process
- Describe the types of decision-making environments
- Use Bayes theorem to establish posterior probabilities
- Make decisions under uncertainty
- Use probability values to make decisions under risk
- Develop accurate and useful decision trees
- Revise probabilities using Bayesian analysis
- Use computers to solve basic decision-making problems
- Understand the importance and use of utility theory in decision making

Descriptive and Predictive Statistics

- **Descriptive statistics** quantitatively describe the main features of a collection of data (mean, variance, etc.)

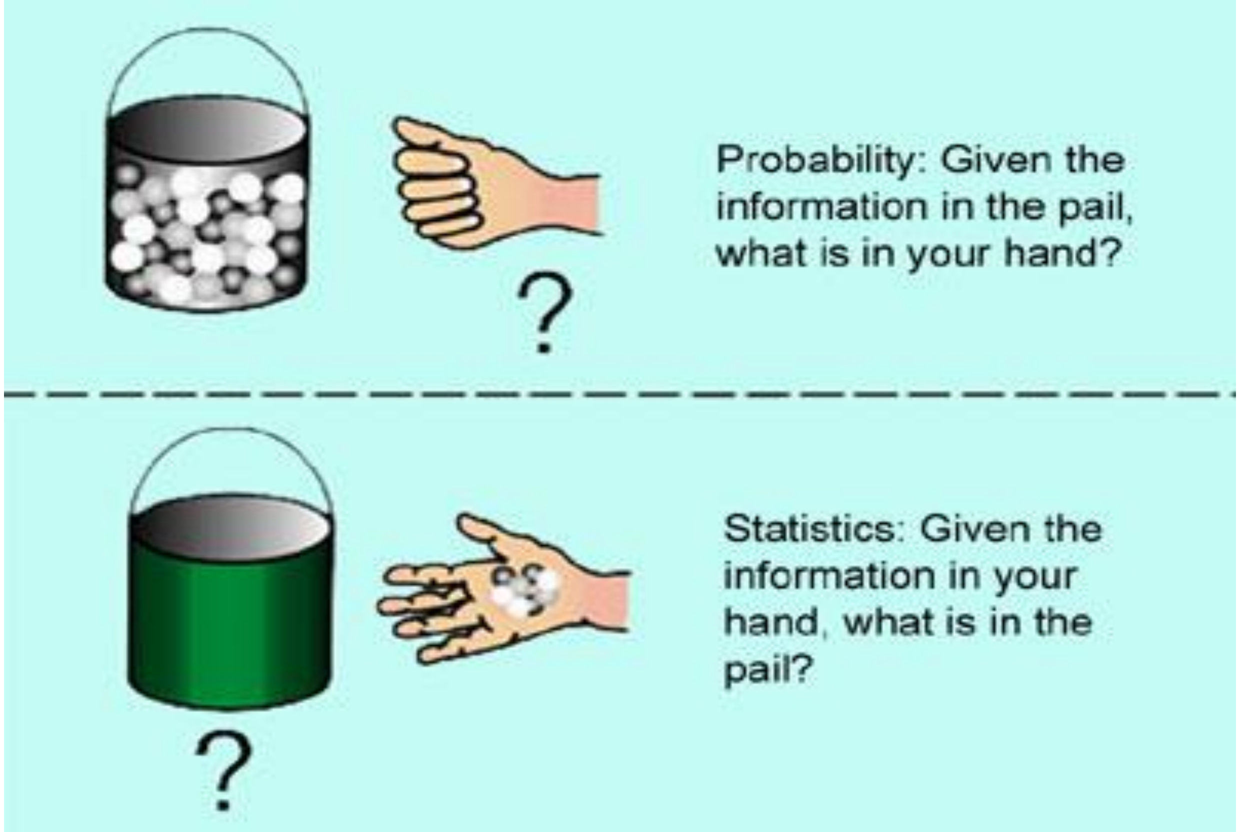
- **Inferential** statistics (or **predictive or inductive** statistics) use sample data to learn about the population that the data represents
  - Proving (supporting) or disproving a hypothesis

- What is the difference between statistics (inferential) and probability ?

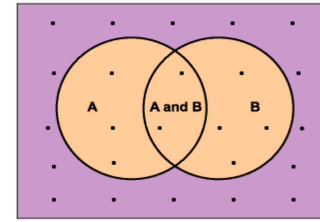


- Do not look ahead !

Probability & Statistics

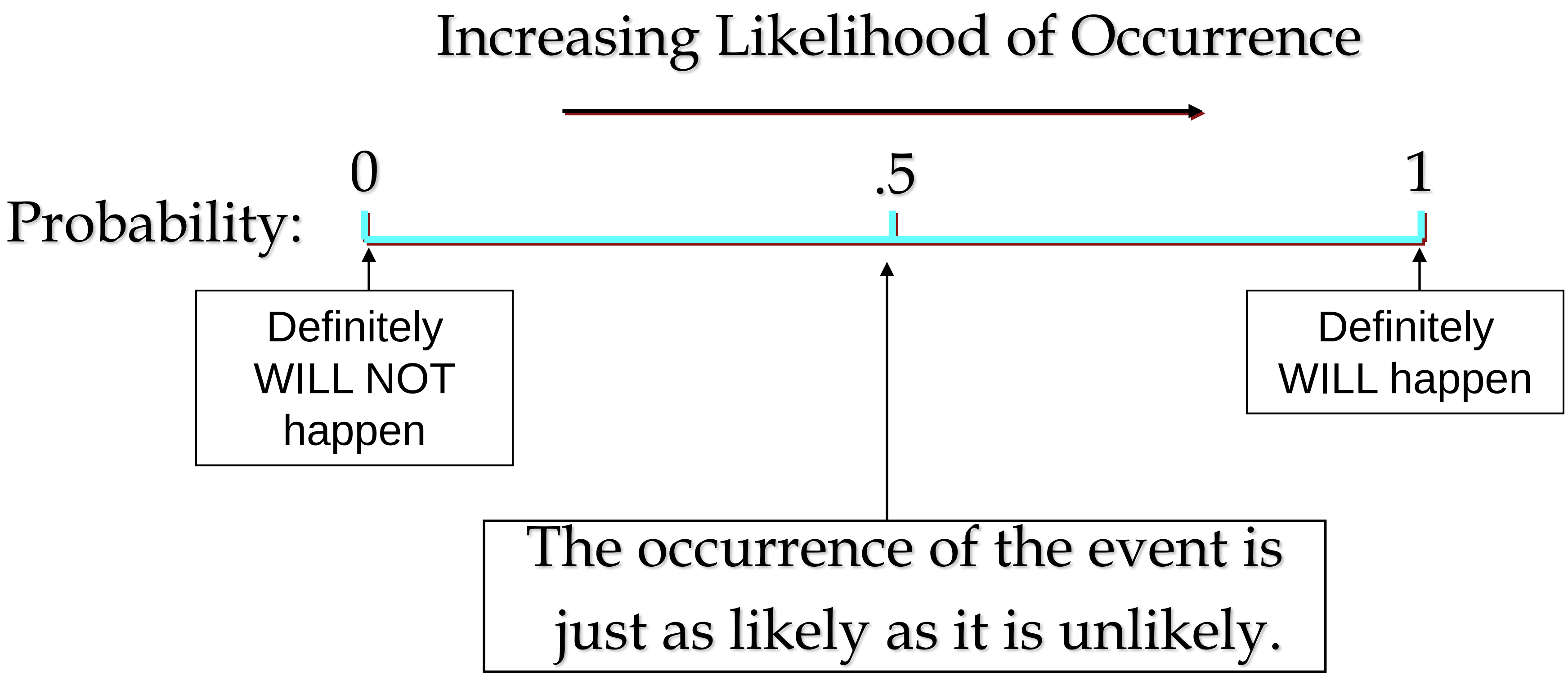


Essential Probability Review



- Probability is a numerical measure of the likelihood that an event will occur
- Probabilities are expressed as percentages that range from 0 to 1 (100 %)
  - A probability near 0 indicates an event is very unlikely to occur
  - A probability near 1 indicates an event is almost certain to occur
  - A probability of 0.5 indicates the occurrence of the event is just as likely as it is unlikely
- Probabilities can be used to express the *degree of uncertainty of an event*

Probability as a Numerical Measure of the Likelihood of Occurrence



Assigning Probabilities To Events

- A probability assigned to a particular outcome must be between 0 and 1 inclusively:

  $0 <= P(E_i) <= 1$ for all $_i$

- The probabilities assigned to each possible outcome of an event must sum to 1.0:

  $P(E_1) + P(E_2) + P(E_3)+\ldots.. + P(E_n) = 1$

Methods For Assigning Probabilities To Events

- Classical (or logical) Method

  Assigning probabilities based on the assumption of <u>equally likely outcomes</u>

- Relative Frequency Method (Objective Method)

  Assigning probabilities based on <u>experimentation or historical data</u>

- Subjective Method

  Assigning probabilities based on the <u>assignor's judgment</u>

Classical (logical) Method

- If an experiment has *n* possible outcomes, this method would assign a probability of 1/*n* to each outcome
- Example:

  Experiment: Rolling a die

  Sample Space: *S* = {1, 2, 3, 4, 5, 6}

  Probabilities: Each sample point has a 1/6 chance (equal chance) of occurring

What is the probability?

- If you flip a coin five times, what is the probability of getting 5 heads?

- Hint: 1 outcome out of total sample space

Exercise (con't)

- Do not look ahead !

Exercise (con't)

- Since there are 2^5 or 32 possible outcomes (in the sample space)
  - n^t
  - where n is the number of outcomes for one trial (2 outcomes) and t is the number of trials (5)
- and {HHHHH} is one sample point
- there is a 1/32 probability of getting that sample point is:
- 1/32 = .03125 or 3.125%

- What is the probability of rolling a 7 with two dice ?

Hint: how many ways can one roll a 7 out of the total sample space

Exercise (con't)

- Do not look ahead !

Probability Of Rolling A 7 With Two Dice

	first die: 1	2	3	4	5	6
second: 1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

As you can see, there are 36 possible rolls of the two dice. These are all equally probable (P=1/36=0.0278 rounded off). And there are 6 ways to roll 7. The probability of rolling 7 is 6/36 or 0.1667. This illustrates the very basics of probability. When we have y (36) equally likely results, and we want to know the probability that a subset of x (6) of them will occur, then P=x/y. That is the definition of P, the probability that something will happen. The probability that A will happen is written P(A). We can write the probability of rolling a 7 as P(rolling 7)=0.1667.

Relative Frequency Method

- Probabilities are based on past observance of how often an event occurs; and we assume that past frequency will continue into the future
- Example:

$$P(\text{rolling a 6}) = \frac{\text{\# of times a 6 occurred}}{\text{\# of times the dice was rolled}}$$

Example: $115 / 760 \simeq 16.34\%$

Relative Frequency Method (con't)

- During trial tests, 100,000 patients tried a new cholesterol-reducing drug
- 7,000 of them had an adverse reaction
- Therefore, the probability that a person will have an adverse reaction to the drug is 7%

"That number has nothing to do with the lottery or the stock market. That's your cholesterol level."

Subjective Probability Method

- When economic conditions and a company's circumstances change rapidly it might be inappropriate to assign probabilities based solely on historical data
- We can use any data available as well as our experience and intuition, but ultimately a probability value should express our degree of belief that the experimental outcome will occur
- Subjective data may come from: opinion polls, judgment of experts, etc.

Techniques for Forecasting Subjective Probabilities

- *Direct probability forecasting* — an expert is simply asked to estimate the chance that an outcome will occur
- *Odds forecasting* (odds making) — a series of bets are proposed to determine how strongly the bettor(s) feels an event will occur
  - Odds is calculated as x/(1-x)
  - If x = 25% (.25), odds are .25/.75 or 1/3 (1 to 3)
  - if x = 75% (.75), odds are .75/.25 or 3 (3 to 1)
- *Comparison forecasting* — similar to odds forecasting except that one game has known probabilities

Adding Mutually Exclusive (disjoint) Events (OR)

We often want to know whether one or a second event will occur

- When two events are mutually exclusive (can't draw both a spade and a club), the law of addition is –

***P* (event *A* or event *B*) = *P* (event *A*) + *P* (event *B*)**

$$
\begin{aligned}
P\ (\text{spade or club}) &= P\ (\text{spade}) + P\ (\text{club}) \\
&= {}^{13}/_{52} + {}^{13}/_{52} \\
&= {}^{26}/_{52} = {}^{1}/_{2} = 0.50 = 50\%
\end{aligned}
$$

Adding Non Mutually Exclusive (joint) Events (OR)

The equation must be modified to account for double counting

- The probability is reduced by subtracting the chance of both events occurring together

P (event *A* or event *B*) = *P* (event *A*) + *P* (event *B*)
– *P* (event *A* and event *B* both occurring)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\begin{aligned} P(\text{five or diamond}) &= P(\text{five}) + P(\text{diamond}) - P(\text{five and diamond}) \\ &= {}^{4}/_{52} + {}^{13}/_{52} - {}^{1}/_{52} \\ &= {}^{16}/_{52} = {}^{4}/_{13} \end{aligned}$$

Events And Independence

1. **(a) Your education**
   (b) Your income level
   } *Dependent events*

2. **(a) Draw a jack of hearts from a full 52-card deck**
   (b) Draw a jack of clubs from a full 52-card deck
   } *Independent events*

3. **(a) Chicago Cubs win the National League pennant**
   (b) Chicago Cubs win the World Series
   } *Dependent events*

4. **(a) Snow in Santiago, Chile**
   (b) Rain in Tel Aviv, Israel
   } *Independent events*

Summary of Probability Formulas

- Addition (OR) formula:
  - When events are mutually exclusive (disjoint - can't happen together):

$$P(A \text{ or } B) = P(A) + P(B)$$

  - When events are joint (can happen together):

$$P(A \text{ or } B) = P(A) + P(B) - P(A * B)$$

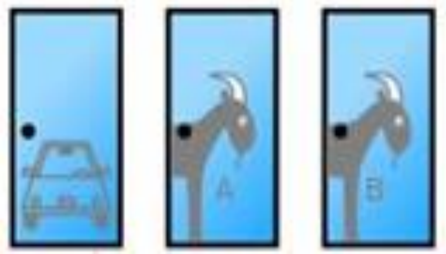
- Multiplication (AND) formula:
  - When events are independent:

$$P(A \text{ and } B) = P(A) * P(B)$$

  - When events are dependent (conditional):

$$P(A \text{ and } B) = P(A) * P(B|A)$$

Trusting Our Intuition ?



- Suppose you're on a game show and you're given the choice of <u>three doors</u>; behind one door is a car; behind the others, goats
- The car and the goats were placed randomly behind the doors before the show
- The rules of the game show are as follows: after you have chosen a door, the door remains closed for the time being – what is the probability that you will chose the car ?
- The game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it

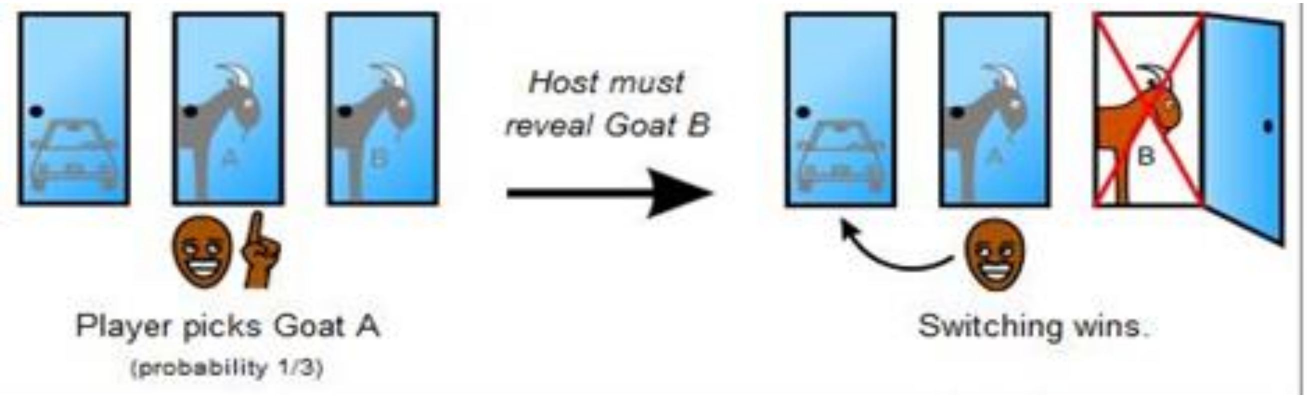
Trusting Our Intuition (con't)

- After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door
- Imagine that you chose Door 1 and the host opens Door 3, which has a goat
- He then asks you "Do you want to switch to Door Number 2?"
- **Is it to your advantage to change your choice?**

- Do not look ahead !

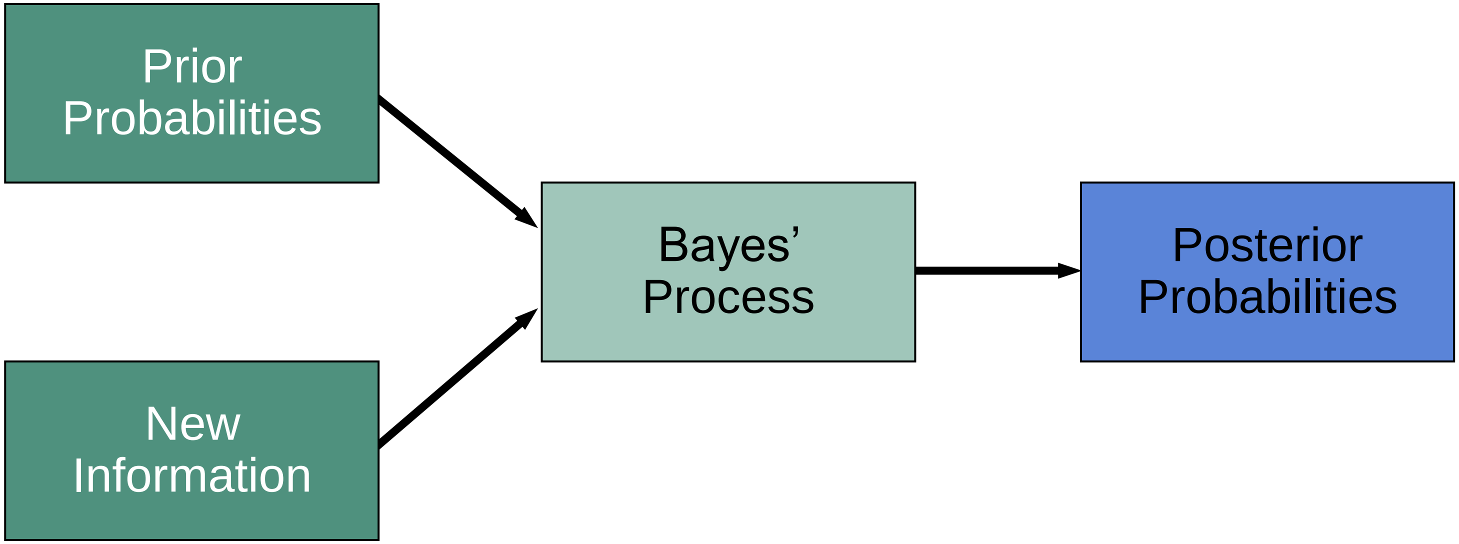
Trusting Our Intuition (con't)

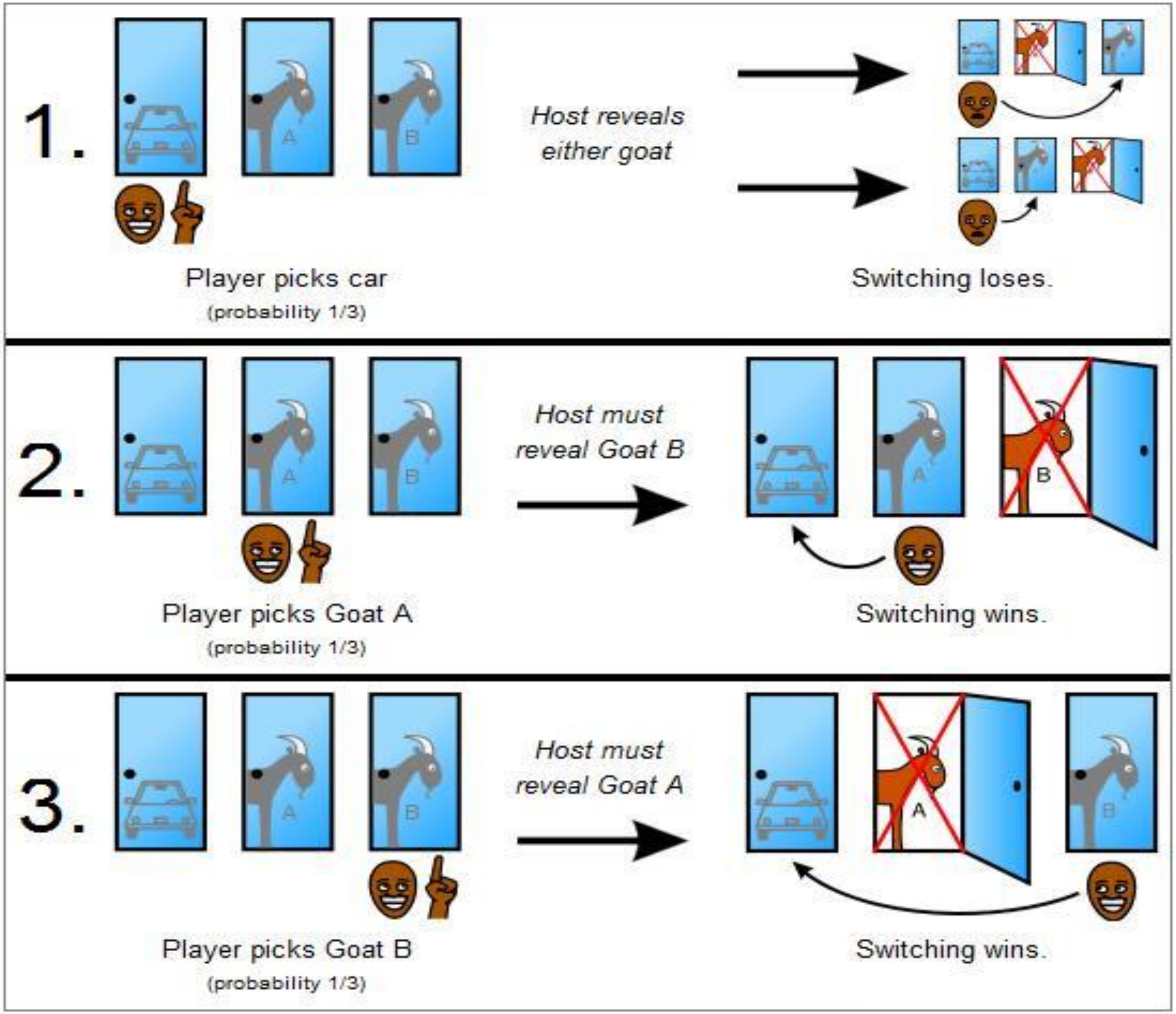
- We started out knowing that there was a 1 in 3 chance of picking a car
- But we now have "additional information" – what door the host chose



The Bayes Rule – New Information

Bayes' theorem is used to incorporate additional information and help create *"posterior probabilities"*





The player has an equal chance of initially selecting the car, Goat A, or Goat B. Switching results in a win 2/3 of the time.

Business Decisions

- Some business decisions can be carried out as though the required information were definitive, such as:
  - The cost is X
  - We will sell Z units per month
  - Development time will be Y months
- But most decisions involve information that is approximate and probabilistic in nature

Types of Decision Making Environments

Type 1: Decision making under certainty

- Decision maker *knows with certainty* the consequences of every alternative or decision choice

Type 2: Decision making under uncertainty

- The decision maker *does not know* the probabilities of the various outcomes

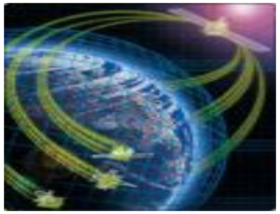
Type 3: Decision making under risk

- The decision maker *knows the probabilities* (or approximate probabilities) of the various outcomes

Key Concepts of Management Decisions

- In a decision, what you do not know about a situation is as important as what you do know, and this uncertainty can be quantified
- Your experience (additional info) modifies your knowledge, and in particular alters the model of your uncertainty

Key Concepts of Management Decisions (con't)



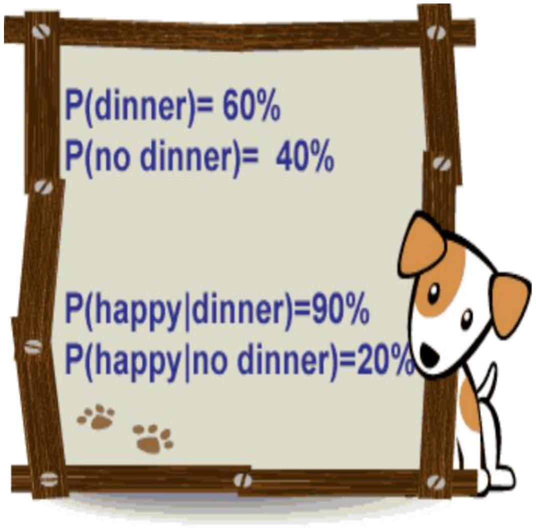
- In some situations it is worthwhile to pay money and/or time to get more information before making a commitment
- And in others the value of the added information is not worth the cost
- And it is possible to calculate whether the cost of the added information is worthwhile

The Bayes Rule

- In the early 18th century an English philosopher and clergyman, Thomas Bayes devoted his attention to a perplexing problem
  - Suppose there is a set of events, one of which must happen
  - It is possible to compute the chance that each will occur through the rules of probability
  - Now suppose that two or more of these events could give rise to the same observation (i.e. one of several ways to roll a 7 with two dice)
  - <u>What is the probability that the observation came from a particular one of the events ?</u>

Decomposing Complex Probabilities

- Probabilities for complex events may be more easily generated by using <u>conditional probabilities</u> within subsets of the events
- For example, it may be easier to forecast sales of a weather-related product (i.e. umbrella) by forecasting sales under good weather, then bad weather; and then considering the probability of bad weather



The Bayes Rule (con't)

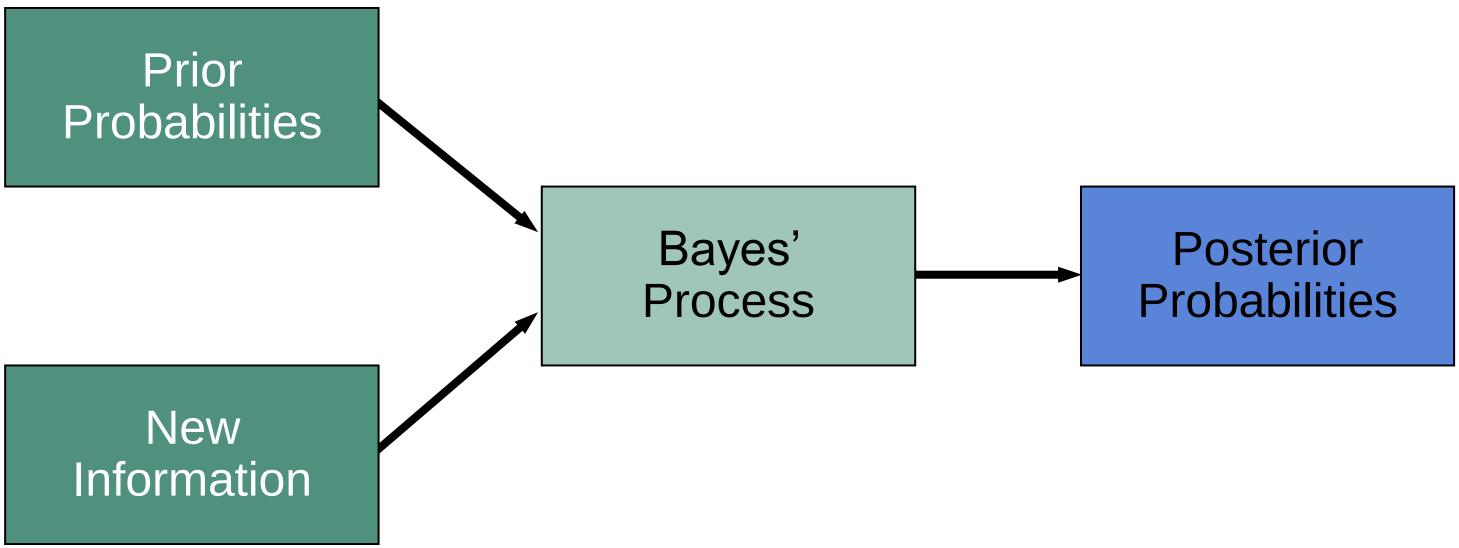
- Suppose there is a set of events, one of which must happen
  - It must rain, snow, or be dry (no precip)
- We're at the time of year where any of these events is possible (i.e. November)
- It is possible to compute the chance that each will occur through the rules of probability
  - Historically for this time of year: P(R) = .3, P(S) = .2, P(D) = .5 (known probabilities)

The Bayes Rule (con't)

- Now suppose that two or more of these events could give rise to the same observation
  - We have noted that the relative humidity today is 100% (additional info)
- What is the probability that the observation came from a particular one of the events ?
  - **Such as, what is the probability that it is snowing ?**

The Bayes Rule (con't)

Bayes' theorem is used to incorporate additional information and help create *"posterior probabilities"*

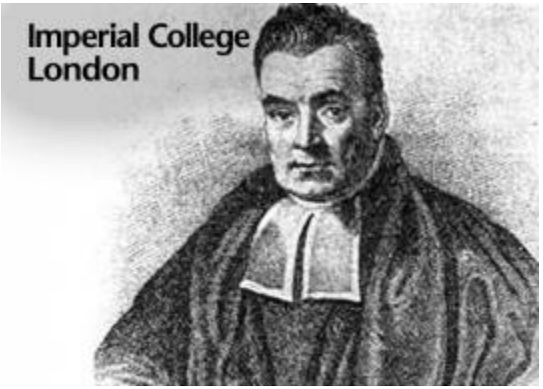


The Bayes Rule (con't)

- Bayes was so unsure of his "rule" that he did not publish it
- His results were published by a friend, Richard Price in 1763
- The same results were later verified by the famous French mathematician Marquis de Pierre Simom LaPlace in 1774

Derivation of Bayes Rule

- For any two events A and B, the probability that they <u>both</u> will occur is P(AB) {P of A and B}
- From the definition of conditional probability:
  - $P(AB) = P(A)P(B|A)$ and $P(AB) = P(B)P(A|B)$
- Thus (since both lhs are the same):
  - $P(A|B) = P(A)P(B|A)/P(B)$



Derivation of Bayes Rule (con't)

- Furthermore, if the event B could have been observed (additional info) for several different events of A (a's):
  - Ie. 100% humidity could be observed by either rain or snow
  - $P(B) = \Sigma P(a)P(B|a)$ where the sum is over the a's
- Thus [by substituting the above formula for P(B) into the previous formula] the form of Bayes Rule relevant for decision analysis is:
  - **$P(A|B) = P(A)P(B|A)/(\Sigma P(a)P(B|a))$**

Meaning of Bayes Rule

- $P(A|B) = P(A)P(B|A)/(\Sigma P(a)P(B|a)$
- We work "backwards" to get $P(A|B)$ by observing $P(a)P(B|a)$ knowing $P(A)$ and $P(B|A)$
- In other words: If we observe that the event B occurred, we modify our statement about the probability that A occurred by the ratio of the probability that B would occur for a particular A to the total probability that B would occur in all the ways that it might have occurred
- It is immaterial whether we speak of "occurred" or "will occur"

Probability of Snow

- What is the probability of snow (A) given that the humidity is 100% (B)
- $P(A|B) = P(A)P(B|A)/(\Sigma P(a)P(B|a)$
- $P(A)$ = probability of snow = .2
- $P(B|A)$ = probability of 100% humidity given that it is snowing = 1
- $\Sigma P(a)P(B|a) = .5$ (two events could yield 100% humidity)
  - $P(a_1)P(B|a_1) = .3$ for rain (prob of rain times the probability of 100% humidity with rain)
  - $P(a_2)P(B|a_2) = .2$ for snow (prob of snow times the probability of 100% humidity with snow)
- $P(A|B) = .2(1)/.5 = 40\%$

Another Example of Bayes Rule

- There are two canvas bags full of poker chips which are identical except for the color of chips within
- The first bag has 30 white chips and 70 green chips, and we shall call this bag the predominantly green bag (G)
- The second bag has 70 white chips and 30 green, and we shall call this bag the predominantly white bag (W)
- We now mix up the two bags so that one does not know which is which
- What is the probability that you selected the G bag ?
- We are concerned with one's judgments about whether a selected bag is the G or W bag

Two Bags of Chips

[the bags are “mixed up”, so we do not know which one was selected]

70 green
30 white

Green Bag

70 white
30 green

White Bag

Two Bags of Chips (con't)

- Now one draws 12 chips at random (one at a time with replacement) – additional info
- It turns out that one drew 8 green chips and 4 white chips
- What are the odds that the bag that was drawn from was the G bag ?

Two Bags of Chips (con't)

- Define event G = predominantly green bag selected
- Define event W = predominantly white bag selected
- P(G) = P(W) = 0.5 (either bag could have been picked)
- Let <u>event A</u> = g g w g w g g w g w g g (the order is unimportant, only the ratio of 2 g's per w)
- Known probabilities:
  - P(g|G) = 0.7 and P(g|W) = 0.3
  - P(w|W) = 0.7 and P(w|G) = 0.3

Two Bags of Chips (con't)

- Probability of the drawing A from the G bag is:
  - $P(A|G) = (0.7)^8(0.3)^4 = 0.0002347$
- Probability of drawing A from the W bag is:
  - $P(A|W) = (0.3)^8(0.7)^4 = 0.000007875$
- From Bayes Rules:
  - $P(G|A) = P(A|G)P(G)/(P(A|G)P(G) + P(A|W)P(W))$
  - $P(G|A) = 0.9674$
- Thus we "work back" from P(A|G) and P(A|W) to get P(G|A)

Analysis Trees

- Analysis Trees represent a graphical interpretation of Bayes Rule
- Again the definition of conditional probability:
  - $P(AB) = P(B)P(A|B)$
- If A can occur if B does or does not (B' is not B):
  - $P(AB') = P(B')P(A|B')$
- Then since A can occur either way (or condition):
  - $P(A) = P(AB) + P(AB')$
  - $P(A) = P(A|B)*P(B) + P(A|B')*P(B')$

Analysis Trees (con't)

- There may be a sever winter or not
  - Let sever winter be event B
  - The probability of B is 0.7
- Let event A be selling of over X units of product (i.e. a heavy winter coat)
- If the winter is sever, the probability of selling over X units of product is 0.8
- If the winter is not sever, the probability of selling over X units of product is 0.5
- What is the probability of selling over X units –> P(A)

Formula Solution

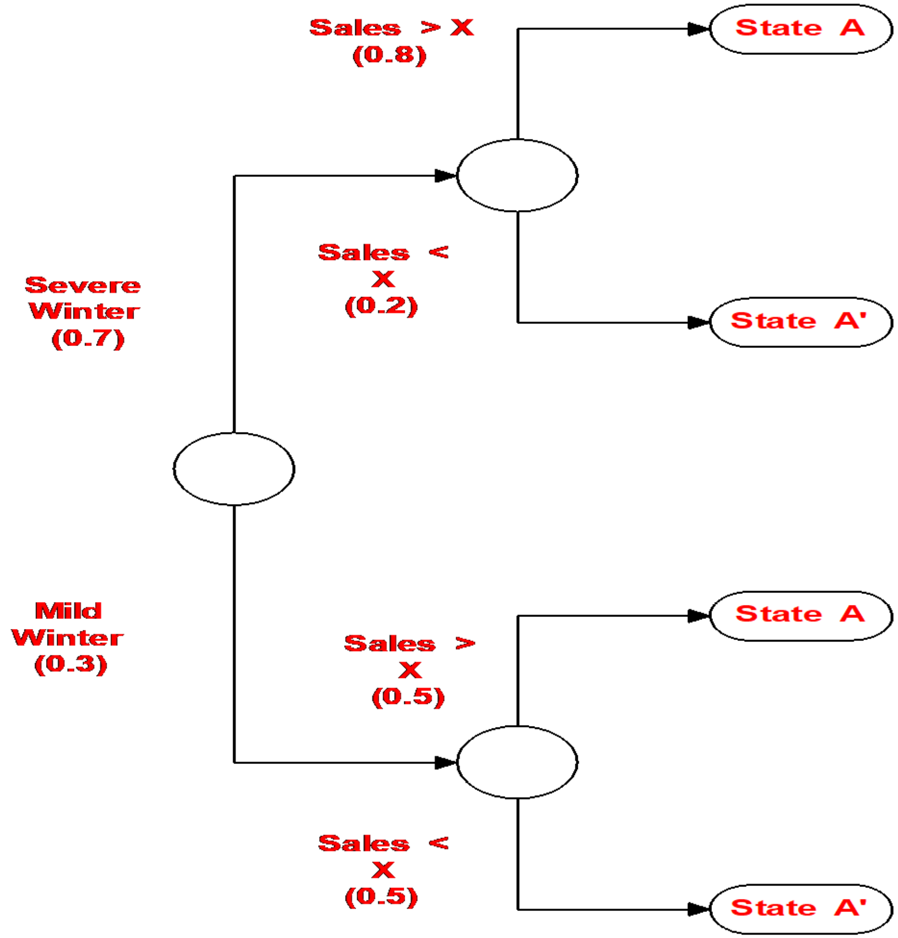
- P(A) = P(A|B)*P(B) + P(A|B')*P(B')
- Now P(A|B') = 1 – P(A|B)
- And P(B') = 1 – P(B)
- P(A) = (0.8 * 0.7) + (0.5 * 0.3) = 0.71

P(A|B) P(A|B')

sever winter mild winter

Graphical Solution

[note the Baysian concept of working backwards]

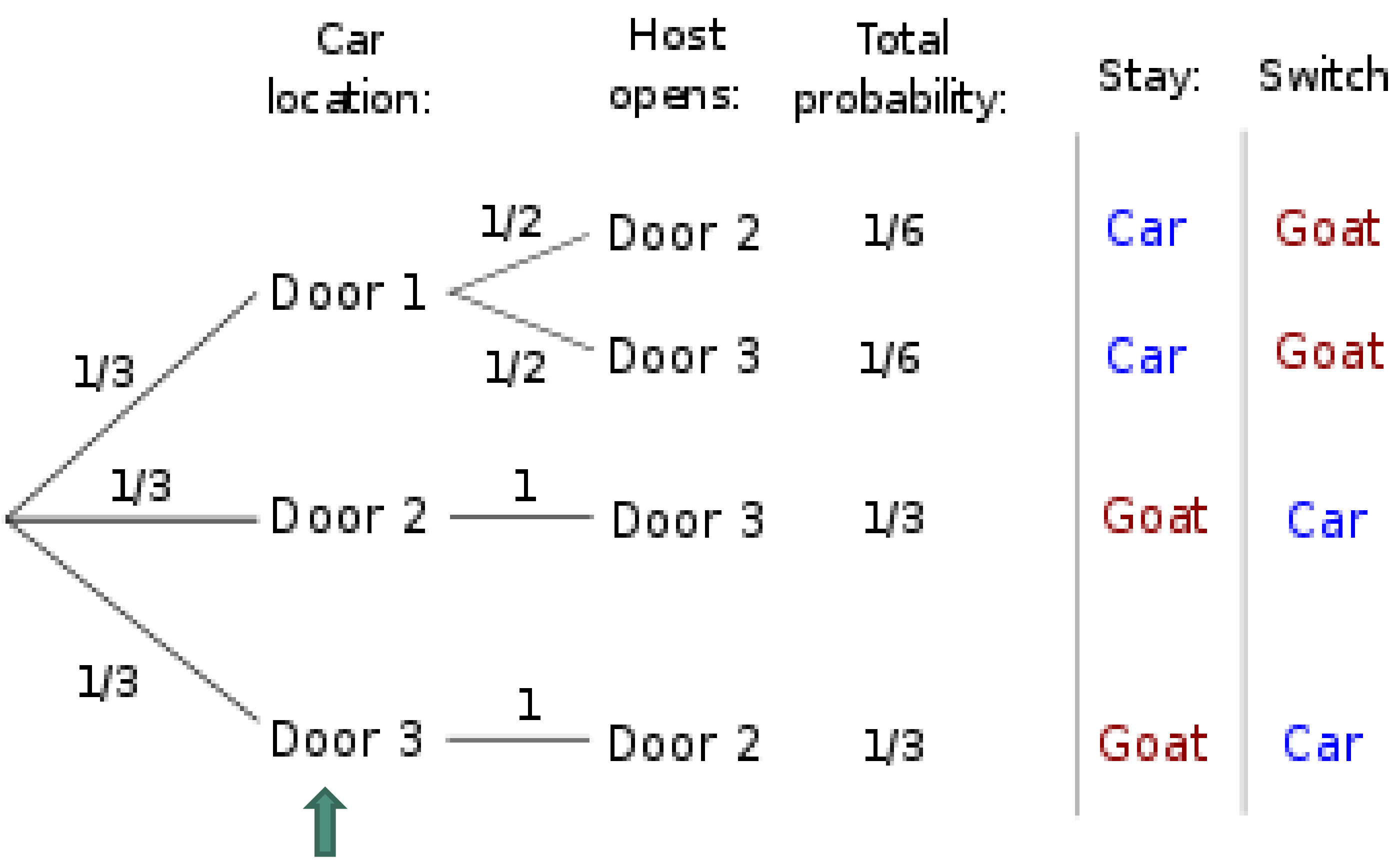


P(A|B) = .8*.7
P(A|B) = .56

P(A|B’) = .5*.3
P(A|B’) = .15

P(A) = .71

Decision Tree for “Monte Hall” Game [car is behind door 1]

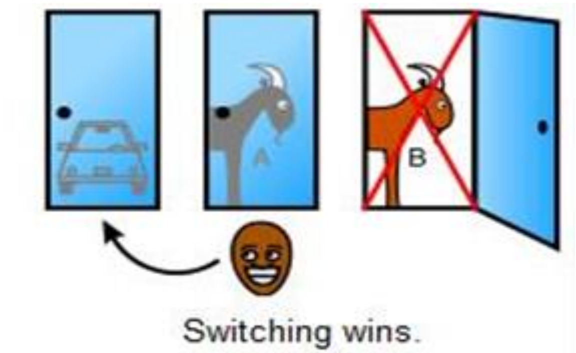


Player chooses

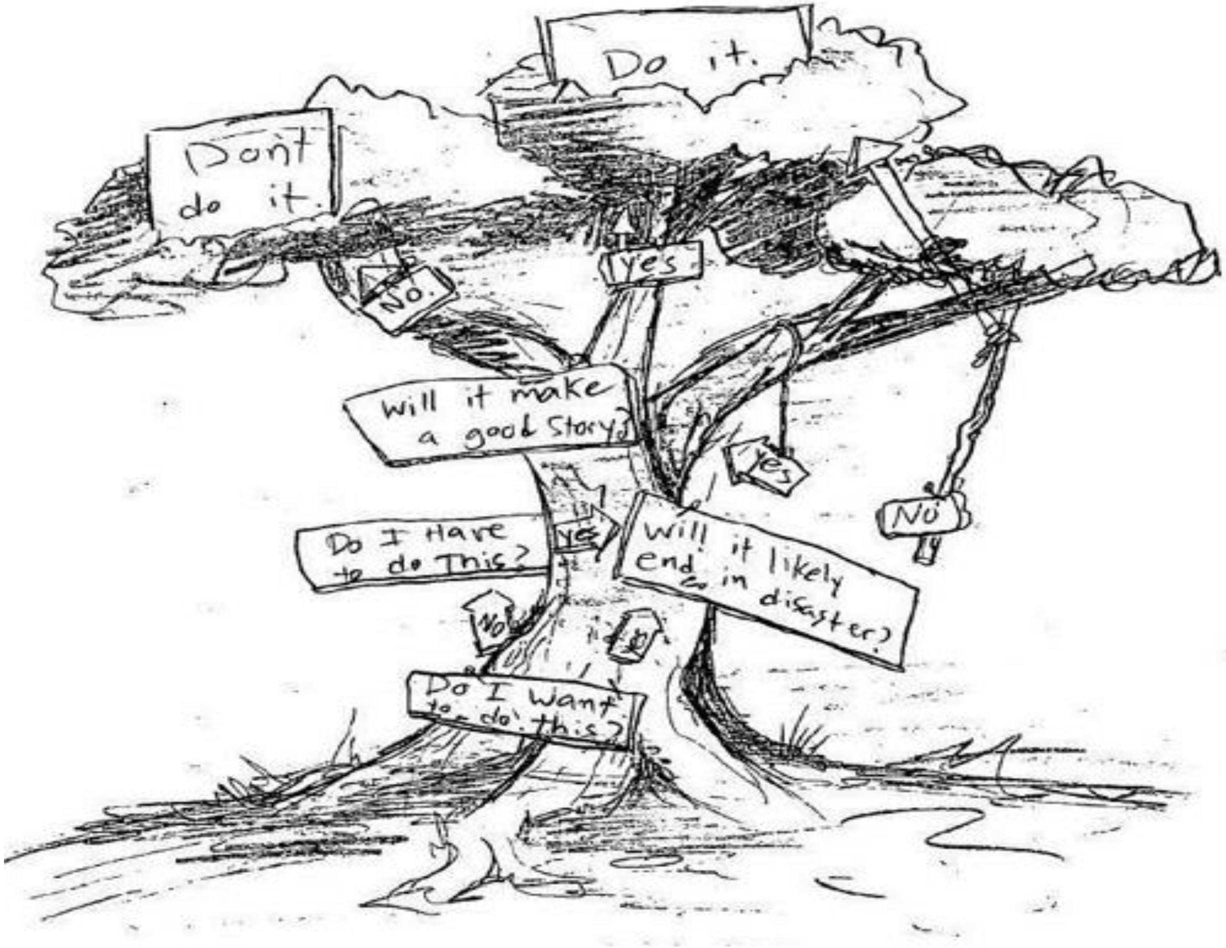
Conditional Probability Example

- P (B|A) = P(AB)/P(A)
  - You chose door 2
  - Find probability that door 2 is goat if host opened door 3 with goat
  - P(2 is goat | 3 is goat) = P (2 is goat and 3 is goat)/P(3 is goat)
  - = (2/3 * 2/3)/ (2/3) = (4/9)/ (2/3) = 12/18 = 2/3
  - Better to switch

Add'l info:
Hosts opened
door 3



Decision Trees



Decision Trees

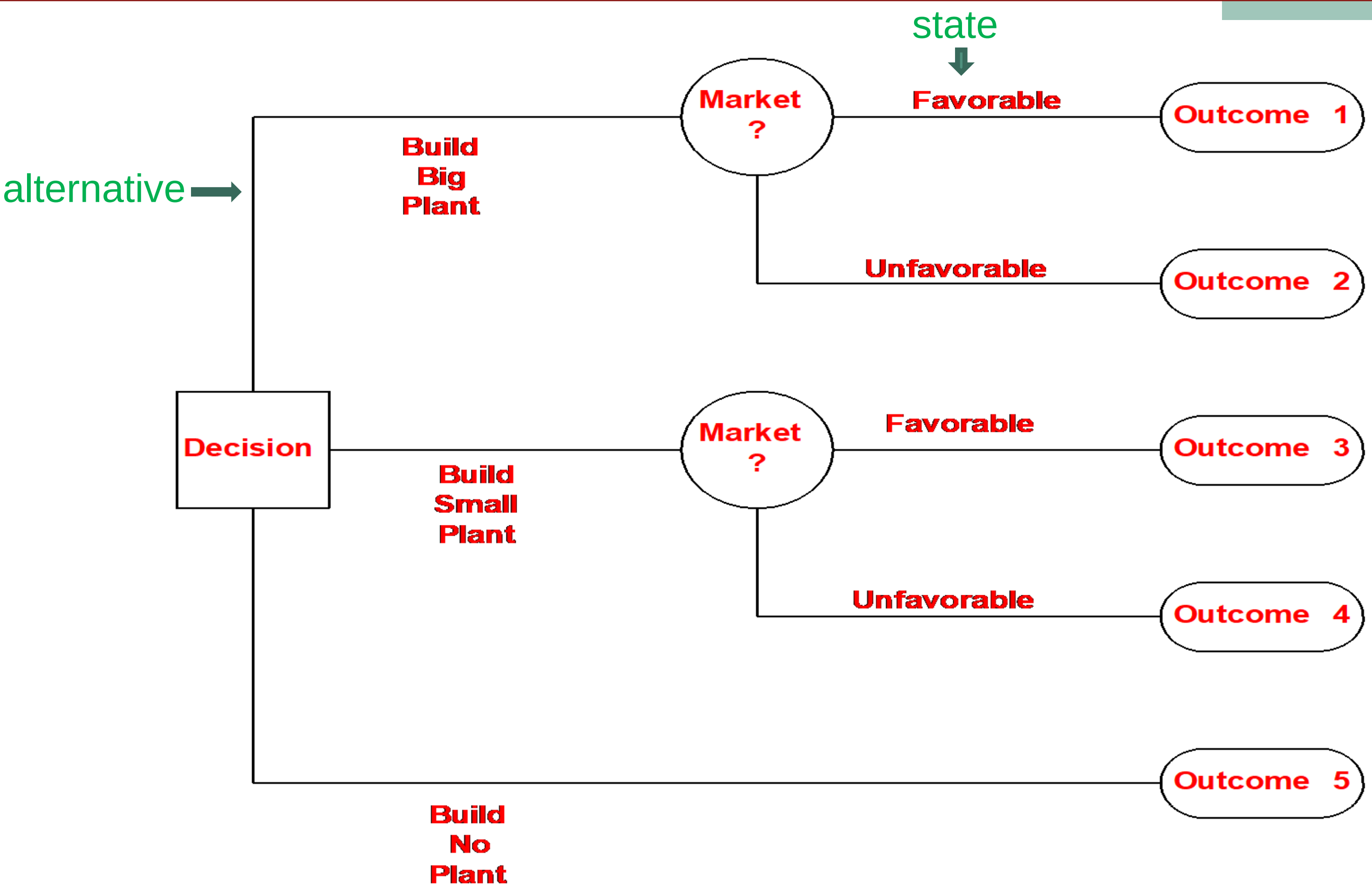
- Decision Trees (payoff trees or payoff tables) are a form of analysis trees used to select a decision based upon some criteria
- In a decision tree there are:
  - “decision nodes” represented by squares
  - “alternatives” represented by lines (arcs) from the decision node
  - “state node” (events over which the decision maker has no control) represented by circles
  - “states” represented by lines from the state node
  - “outcomes” results of decisions and states
- The states may be known in the sense of their likelihood to occur or not
- Outcomes can be calculated qualitatively or quantitatively

Expansion of a Manufacturing Company

- Step 1 – Define the problem
  - Expand by manufacturing and marketing a new product
- Step 2 – List alternatives
  - Construct a large new plant
  - Construct a new small plant
  - Construct no plant at all
- Step 3 – Identify possible outcomes
  - The market could be favorable or unfavorable
- Step 4 – List the payoffs of each combination of alternative & outcome
  - Identify *conditional values* for the profits for large, small, and no plants for each of the two possible market conditions
- Step 5 – Select the decision model
  - Depends on the environment, amount of uncertainty, and your knowledge of that uncertainty
- Step 6 – Apply the model to the data
  - Solution and analysis used to help the decision making

Example Decision Tree

[one can decide which plant to build, but one has no control over the market (state node)]



Which decision to make ?

Decision Table of Outcomes

- A table showing the outcomes (or payoffs) can also be developed
- The rows of the table represent the decisions and the columns represent the states
- For multiple state nodes (levels) there will be columns of columns

	Conditions of Nature (states)		
Alternatives	**Condition 1**	**Condition 2**	**Condition 3**
Alternative 1			
Alternative 2			

Decision Table Outcomes

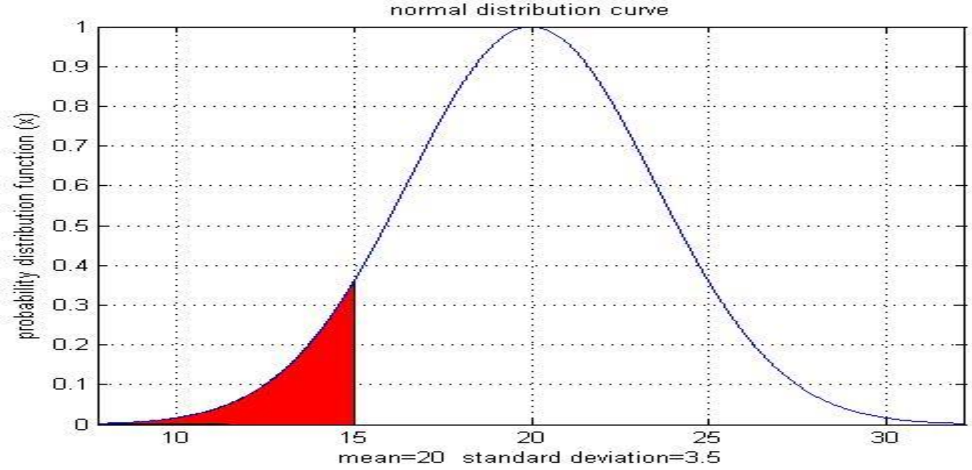
Alternatives	Favorable Market	Unfavorable Market
Big Plant	1000000 (gain 1 million $)	-600000 Loose $600,000
Small Plant	600000	-100000
No Plant	0	0

Which Alternative Should We Chose ?

Alternatives	Favorable Market	Unfavorable Market
Big Plant	1000000	-600000
Small Plant	600000	-100000
No Plant	0	0

- Do not look ahead !

- It depends primarily on the likelihood of the market being favorable or not, and the decision maker's risk tolerance



Decisions Under Uncertainty

- When the probabilities of various states cannot be reliably estimated, then there are techniques to help make a good decision:
  - Maximax – maximize the maximum outcome (brave/risky approach); find the maximum in a row then chose the maximum in that new column
  - Maximin – maximize the minimum outcome (conservative approach); find the minimum in each row then chose the maximim in that new column
  - Equally likely – find the average of each row then chose the maximum of that new column
  - Criterion of realism – weighted average (based on degree of optimism, 0.5 is same as equally likely)
  - Minimax regret – minimize maximum opportunity loss:
    - Cell regret = -(cell value – best in column)
    - Row (alternative) regret = lowest column (state) in each row
    - Minimim regret = lowest cell in regret column

Alternative Selections Under Complete Uncertainty

Alternatives	Favorable Market	Unfavorable Market	Maximax (Brave/Risky) "best of best"	Maximin (conservative) "best of worst"	Equally Likely
Big Plant	1000000	-600000	1000000	-600000	200000
Small Plant	600000	-100000	600000	-100000	250000
No Plant	0	0	0	0	0

Regret

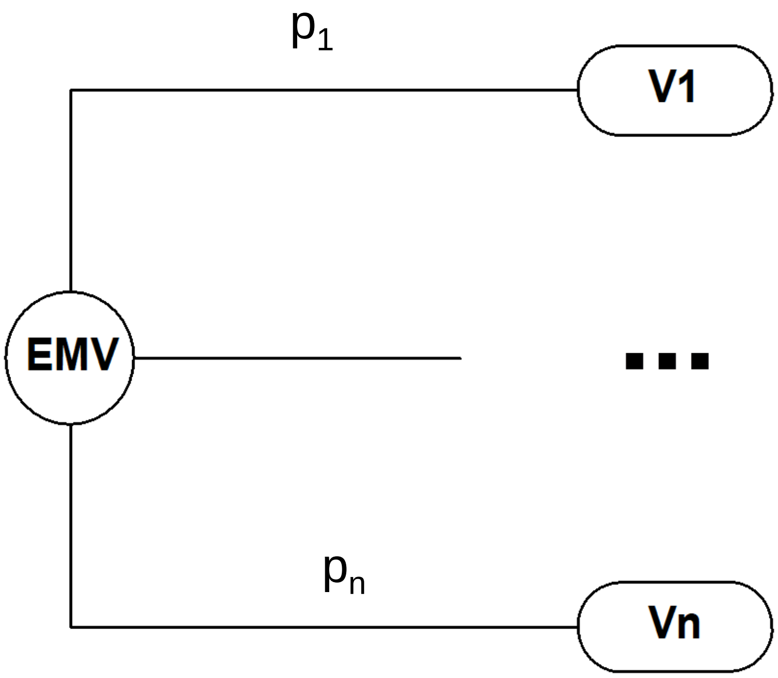
Alternatives	Favorable Market	Unfavorable Market	Worst Regret
Big Plant	1000000 – 1000000 = 0	-600000 – 0 = -600000	-600000
Small Plant	600000 – 1000000= -400000	-100000 – 0 = -100000	-400000
No Plant	0 – 1000000 = -1000000	0 – 0 = 0	-1000000

Decisions Under Risk

- When we can obtain fairly reliable estimates of the probability of each state (for a state node), then it is said the decision is "under risk"
- A EMV (Expected Monetary Value) is calculated for each alternative by multiplying the probability by the monetary outcome
- EMV = Probability of State 1 times the outcome for state 1 + … Probability of State N times the outcome for state N

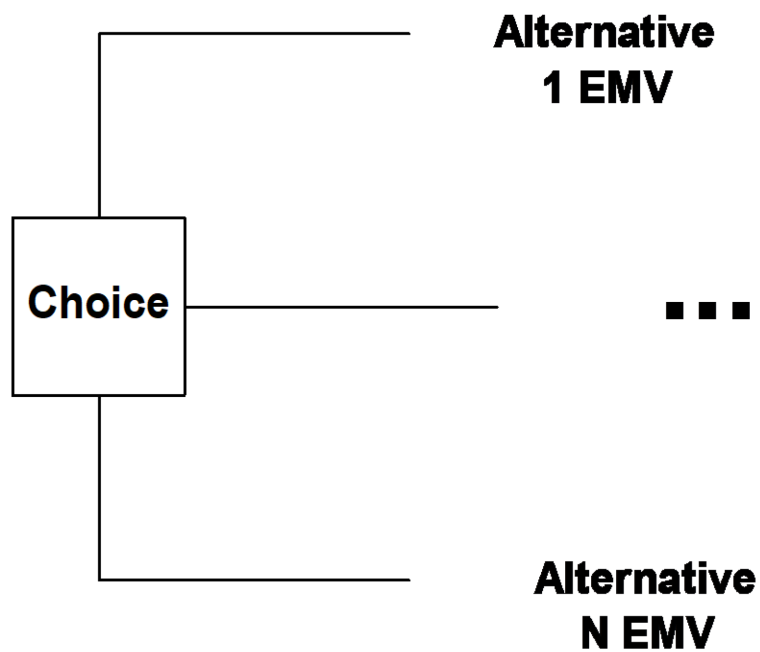
State Node

- $EMV = \Sigma\ p_i\ V_i$



Decision Node

- Choice = Max(EMV_i)



Decisions Under Risk

[50/50 probability of each market condition]

Alternatives	Favorable Market- 50%	Unfavorable Market – 50%	EMV
Big Plant	1000000	-600000	500000 - 300000 = 200000
Small Plant	600000	-100000	300000 – 50000 = 250000
No Plant	0	0	0
Probability	0.5	0.5	

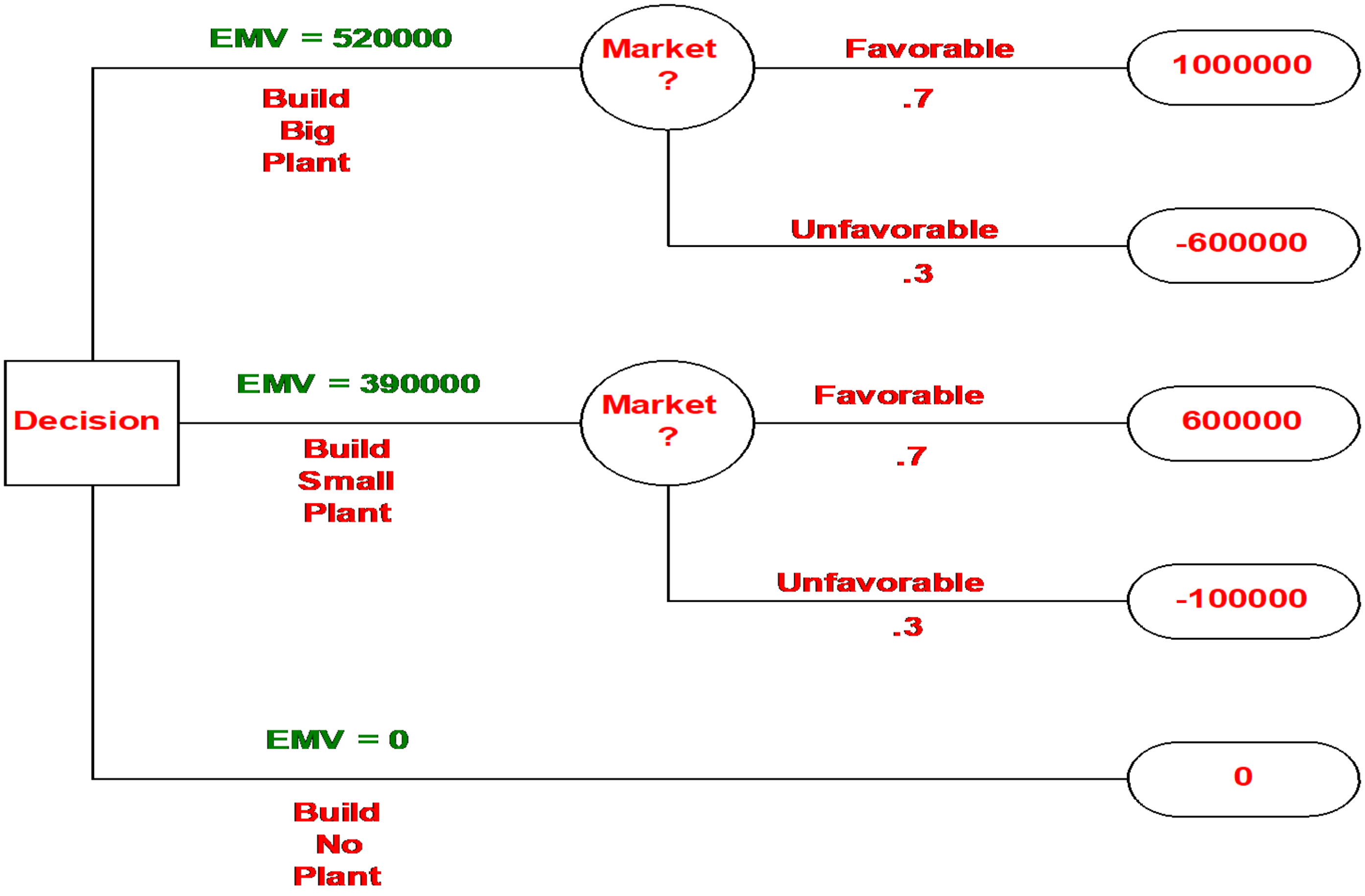
Decisions Under Risk

[70/30 probability]

Alternatives	Favorable Market- 70%	Unfavorable Market – 30%	EMV
Big Plant	1000000	-600000	700000 - 180000 = 520000
Small Plant	600000	-100000	420000 – 30000 = 390000
No Plant	0	0	0
Probability	0.7	0.3	

EMV's on Decision Trees

[70/30 probability]



Sensitivity Analysis

- Sensitivity analysis examines how our decision might change with different input data
- For the current example since there are only two states of nature:

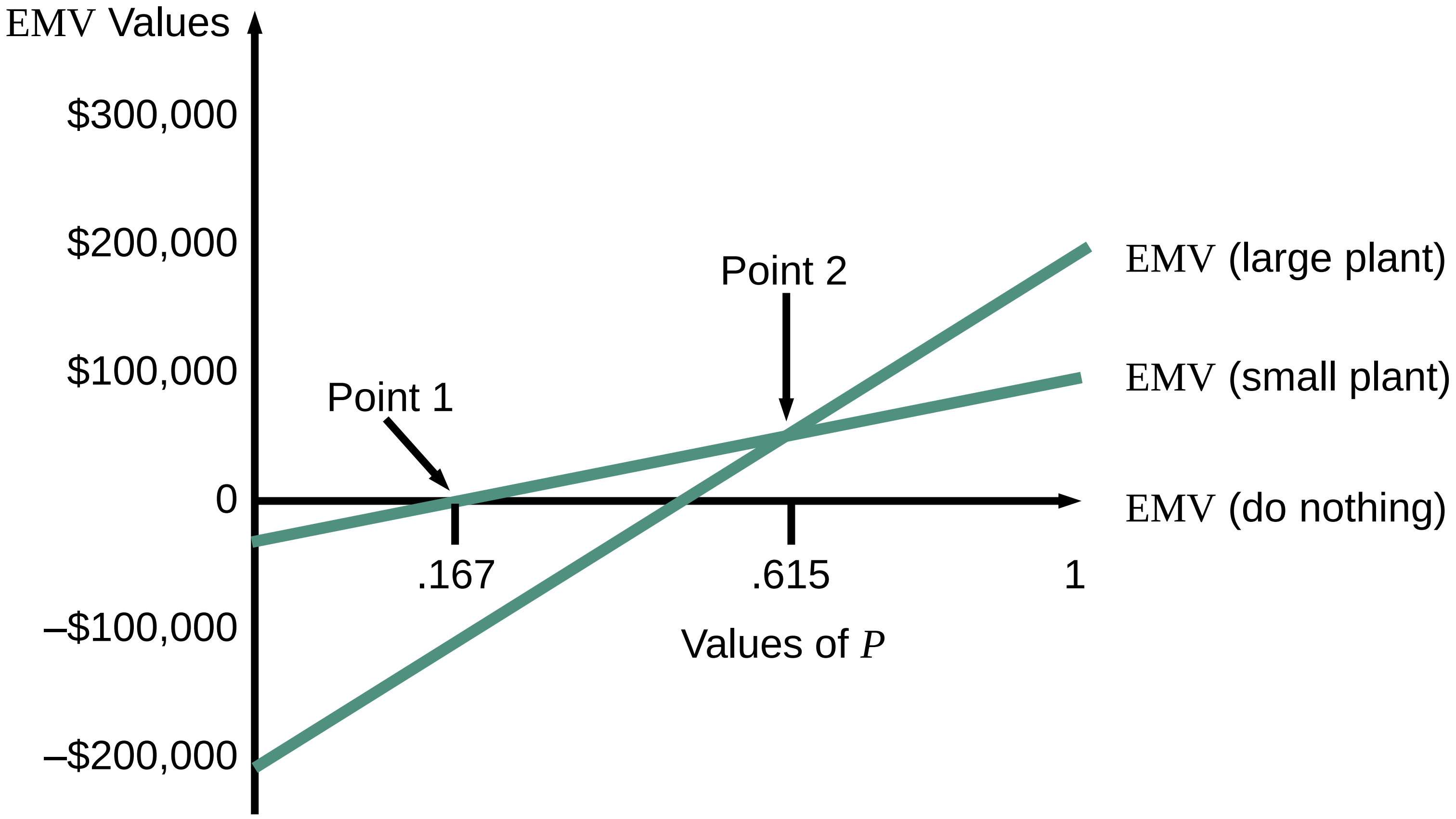
P = probability of a favorable market

$(1 – P)$ = probability of an unfavorable market

Sensitivity Analysis (con't)

EMV(Large Plant)	$= \$200{,}000P - \$180{,}000)(1 - P)$
	$= \$200{,}000P - \$180{,}000 + \$180{,}000P$
	$= \$380{,}000P - \$180{,}000$
EMV(Small Plant)	$= \$100{,}000P - \$20{,}000)(1 - P)$
	$= \$100{,}000P - \$20{,}000 + \$20{,}000P$
	$= \$120{,}000P - \$20{,}000$
EMV(Do Nothing)	$= \$0P + 0(1 - P)$
	$= \$0$

Sensitivity Analysis (con't)



Sensitivity Analysis (con't)

Point 1:

EMV(do nothing) = EMV(small plant)

$$0 = \$120{,}000P - \$20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

Point 2:

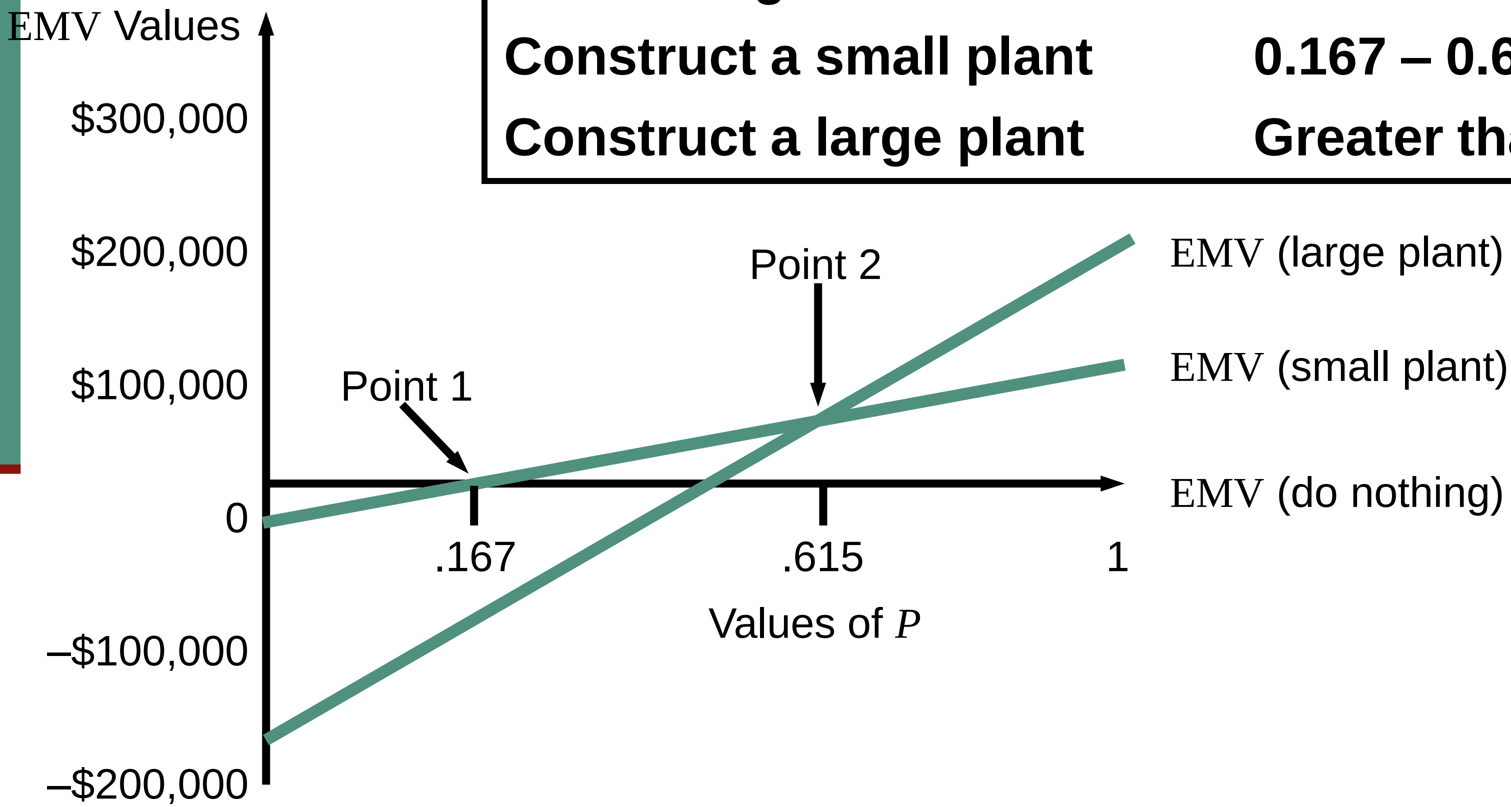
EMV(small plant) = EMV(large plant)

$$\$120{,}000P - \$20{,}000 = \$380{,}000P - \$180{,}000$$

$$P = \frac{160{,}000}{260{,}000} = 0.615$$

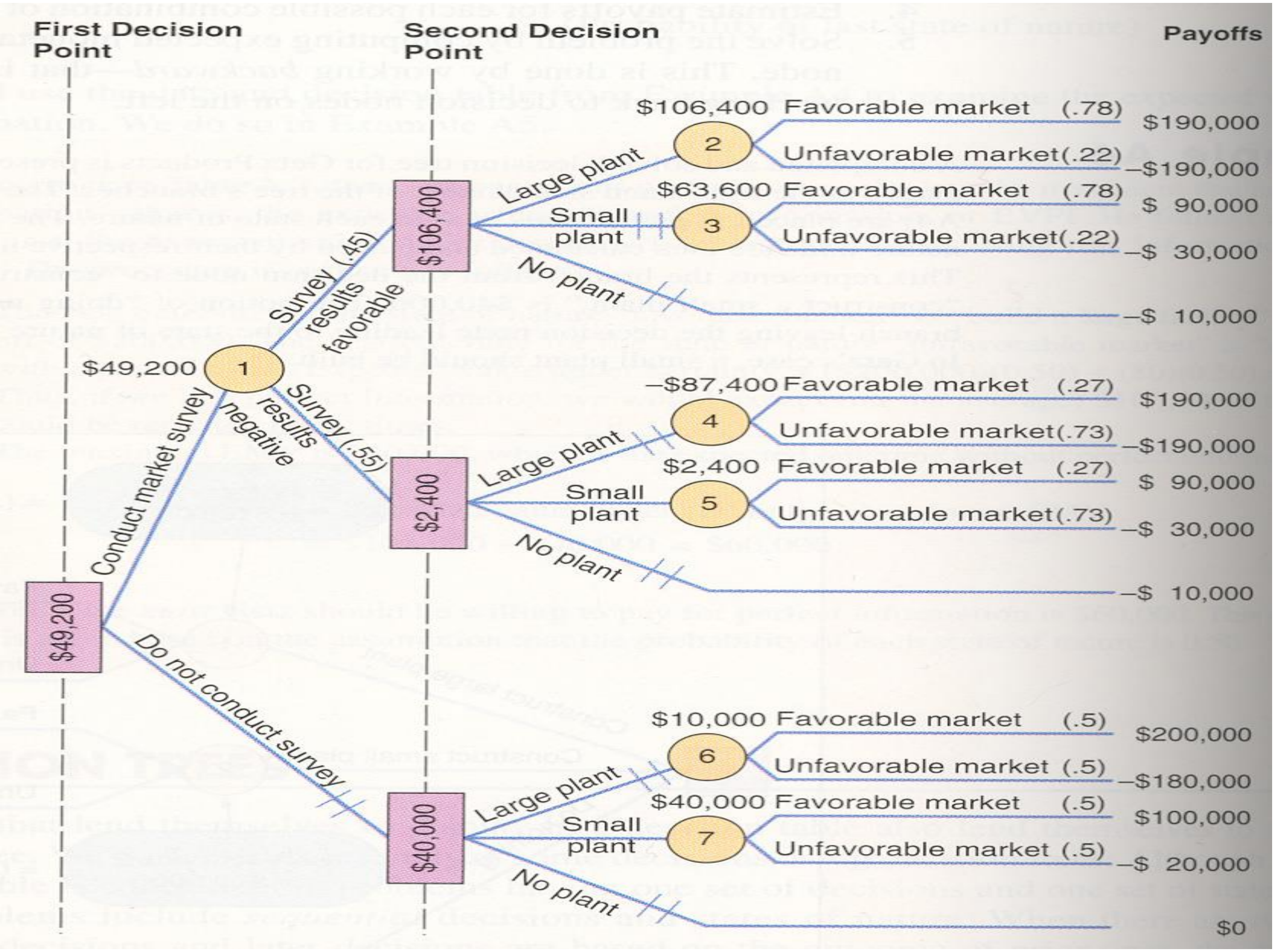
Sensitivity Analysis (con't)

BEST ALTERNATIVE	RANGE OF *P* VALUES
Do nothing	**Less than 0.167**
Construct a small plant	**0.167 – 0.615**
Construct a large plant	**Greater than 0.615**



Multiple Level Decision Trees

[Survey Cost is $10,000]



Expected Value of Perfect Information

[Decisions Under Complete Certainty]

- Suppose that we could pay a price to find out for sure whether the market would be favorable or not
- What is the maximum that we should pay to get this information ?
- An EVPI (Expected Value of Perfect Information) can be determined from the outcomes table
  - EVPI = Expected Value Under Certainty – Maximum EMV
- The Expected Value Under Certainty = **Best Outcome** of State 1 times the Probability of State 1 + … **Best Outcome** of State N times the Probability of State N

Best Outcomes

[for Each of the two states]

Alternatives	Favorable Market	Unfavorable Market
Big Plan	1000000	-600000
Small Plant	600000	-100000
No Plant	0	0

Expected Value of Perfect Information (con't)

- For the prior case of 50/50 probability:
  - Expected Value Under Certainty = 1000000 * .5 + 0 * .5 = 500000
  - Maximum EMV = 250000
  - EVPI = 500000 – 250000 = 250000
- For the prior case of 70/30 probability:
  - Expected Value Under Certainty = 1000000 * .7 + 0 * .5 = 700000
  - Maximum EMV = 520000
  - EVPI = 700000 – 520000 = 180000

Using Excel for Decision Trees

[one can calculate all decision parameters]

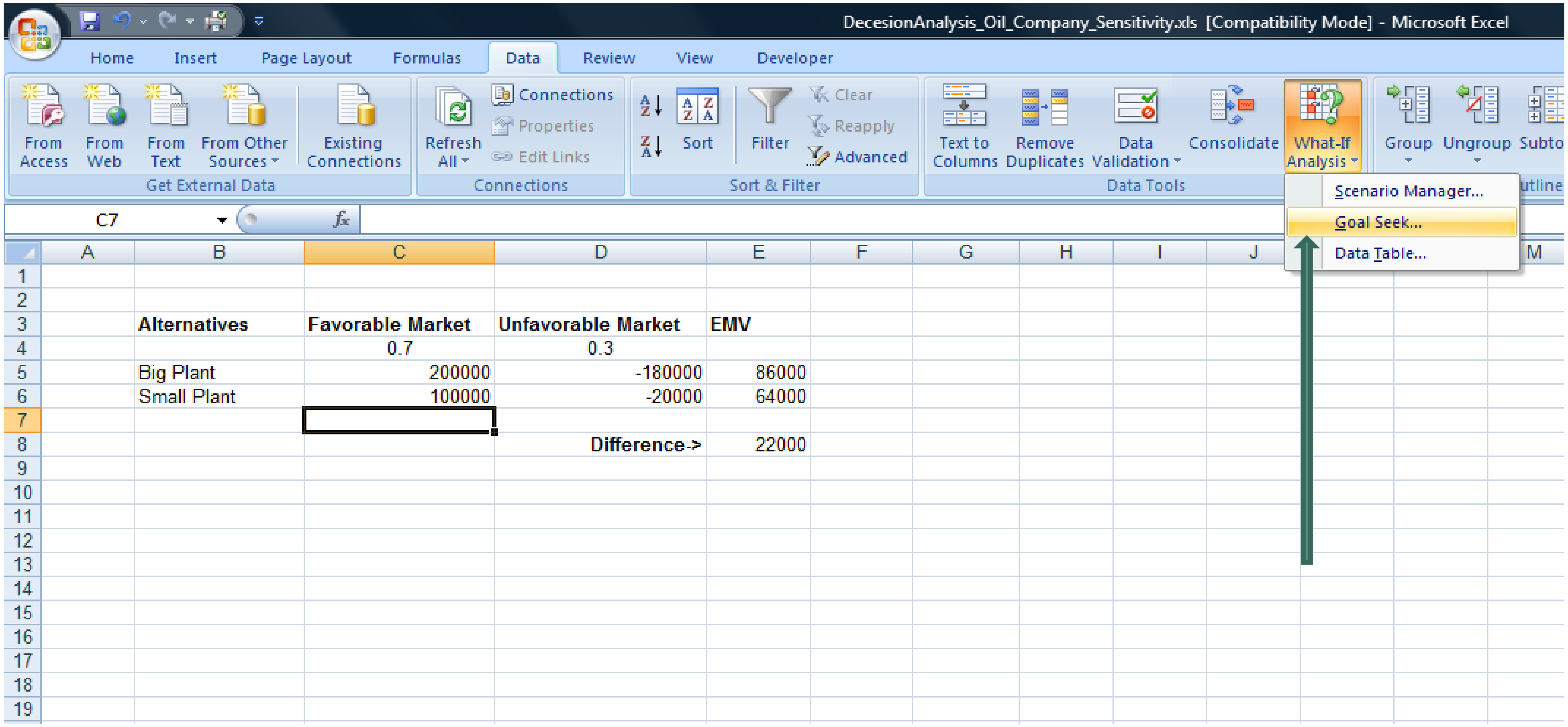
Microsoft Excel - decesionAnalysis.xls

	A	B	C	D	E	F	G	H
1								
2								
3		**Alternatives**	**Favorable Market**	**Unfavorable Market**	**EMV**	**Minimum**	**Maximum**	
4			0.7	0.3				
5		Big Plant	1000000	-600000	520000	-600000	1000000	
6		Small Planmt	600000	-100000	390000	-100000	600000	
7		No Plant	0	0	0	0	0	
8				**Maximum->**	520000	0	1000000	
9								
10		**Column Best**	1000000	0	700000	**<- Expected Value Under Certainty**		
11					180000	**<- EVPI**		
12								
13								

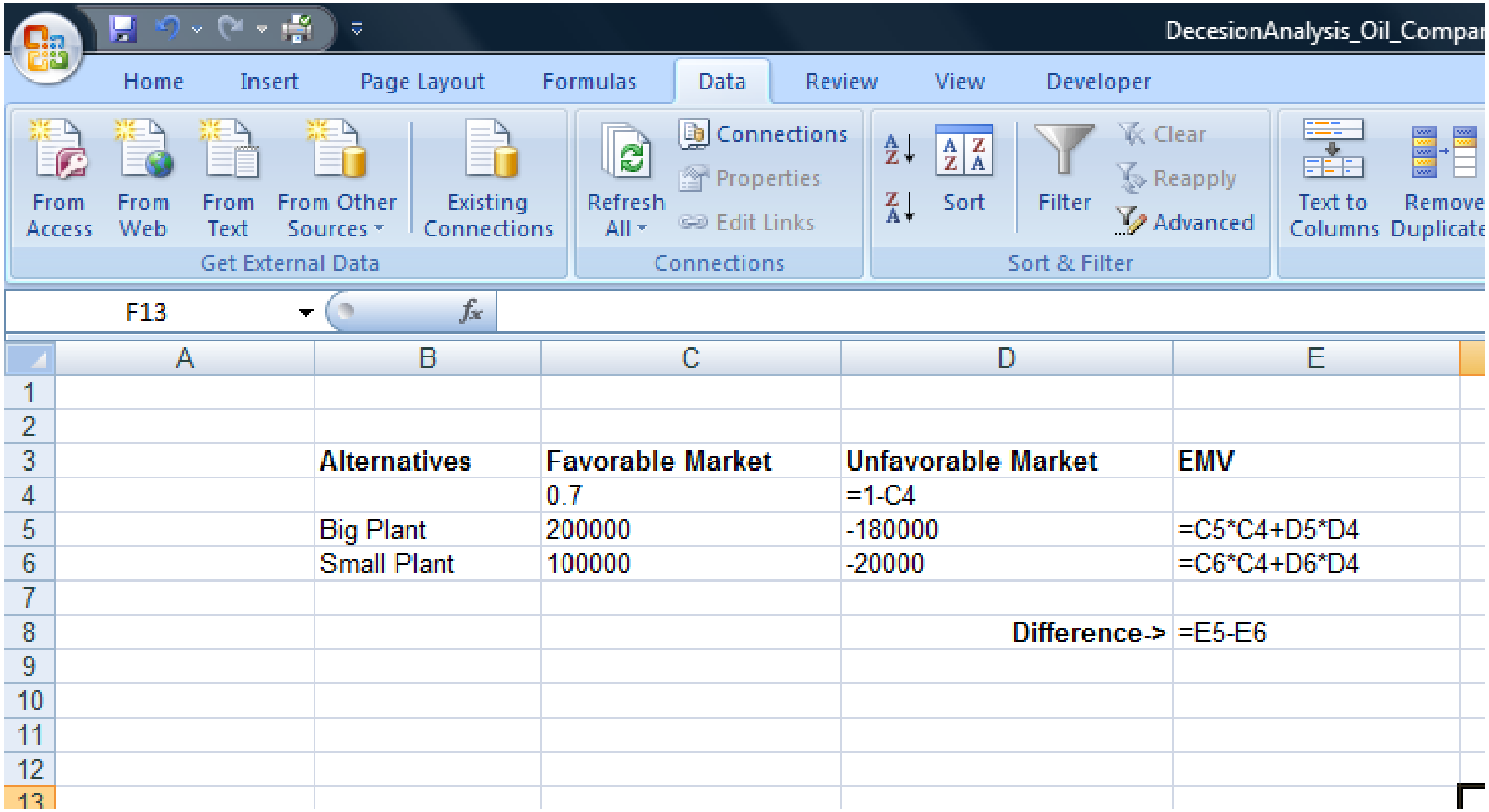
Using Excel for Decision Trees (con't)

B	C	D	E	F	G
Alternatives	**Favorable Market**	**Unfavorable Market**	**EMV**	**Minimum**	**Maximum**
	0.7	=1-C4			
Big Plant	1000000	-600000	=C5*C4+D5*D4	=MIN(C5:D5)	=MAX(C5:D5)
Small Planmt	600000	-100000	=C6*C4+D6*D4	=MIN(C6:D6)	=MAX(C6:D6)
No Plant	0	0	0	=MIN(C7:D7)	=MAX(C7:D7)
		Maximum->	=MAX(E5,E7)	=MAX(F5,F7)	=MAX(G5,G7)
Column Best	=MAX(C5:C7)	=MAX(D5:D7)	=C10*C4+D10*D4	**<- Expected Value Un**	
			=E10-E8	**<- EVPI**	

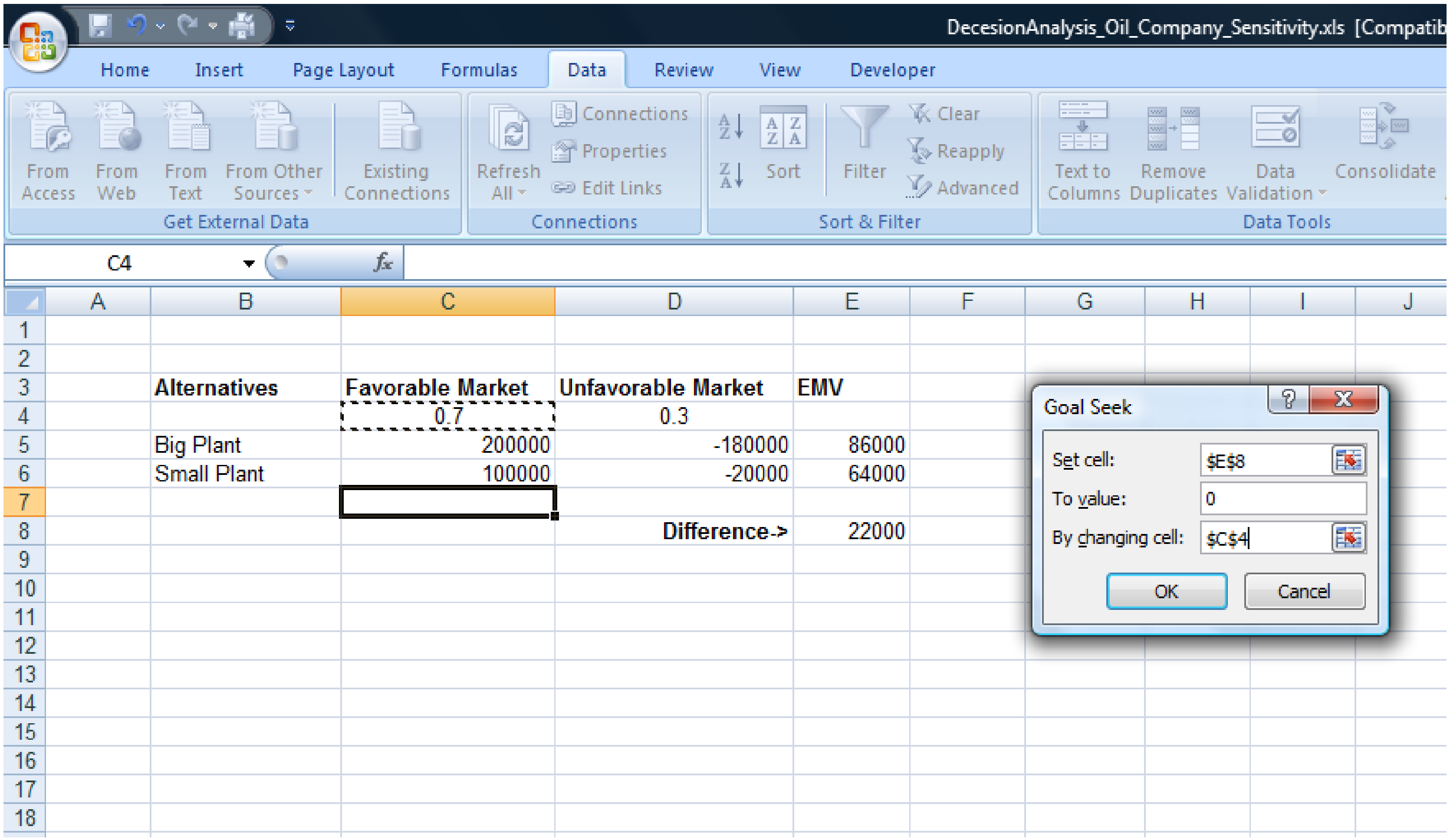
Excel Goal Seek For Sensitivity Analysis



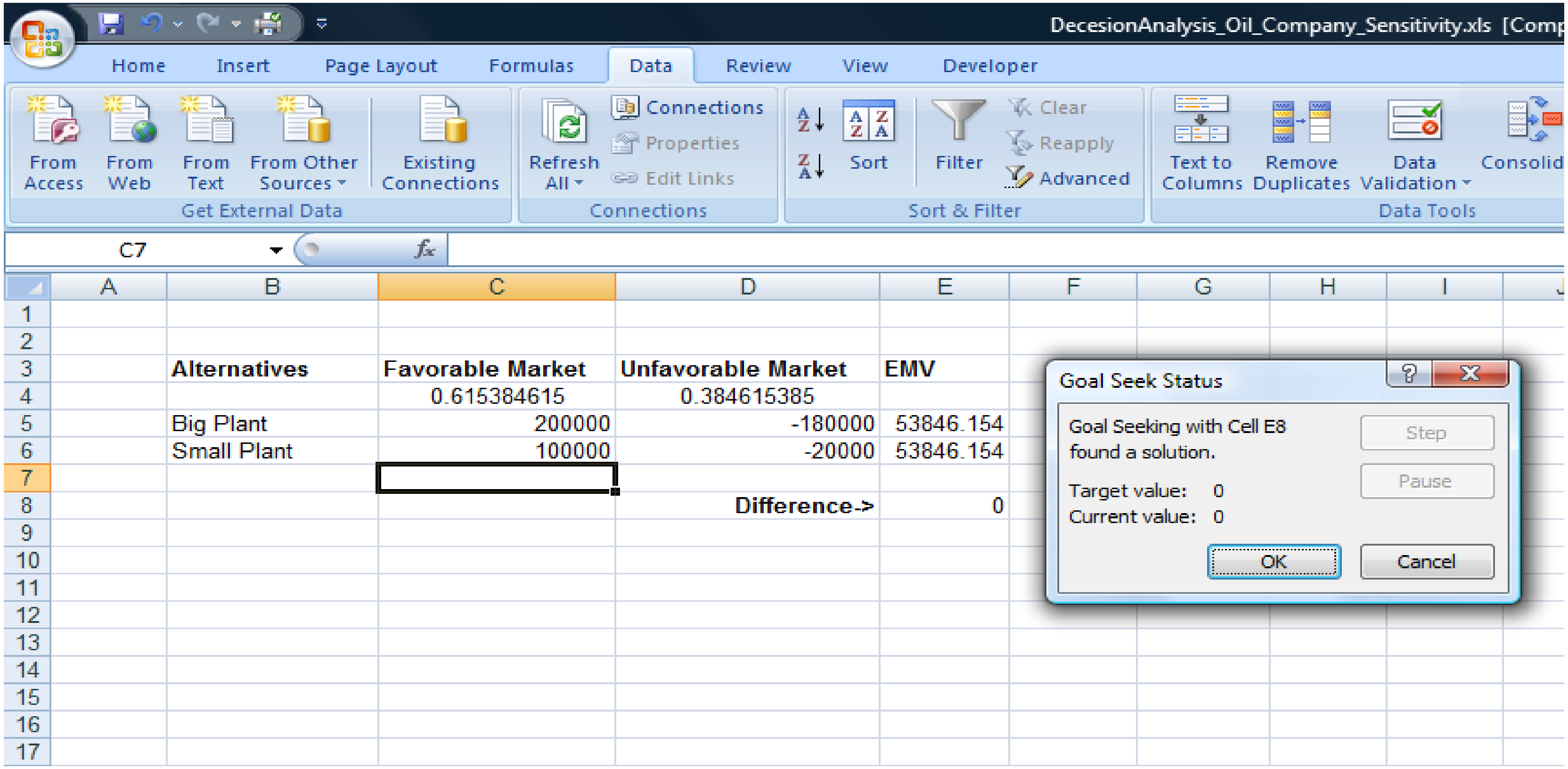
Excel Goal Seek For Sensitivity Analysis - Formulas



Excel Goal Seek For Sensitivity Analysis (con't)



Excel Goal Seek For Sensitivity Analysis (con't)



Excel Lab

- The VP of XYZ Oil Company is considering the purchase of more drilling equipment to expedite drilling operations. His alternatives are:
  - The Mighty Mac
  - The Big Jack
  - The Oiler Otis

- The future market may be favorable or not
- The payoffs for each alternative and market condition are shown on the next slide

Payoff Table

Equipment	Favorable Market ($)	Unfavorable Market ($)
The Mighty Mac	300,000	-200,000
The Big Jack	250,000	-100,000
The Oiler Otis	75,000	-18,000

Excel Lab (con't)

Alternatives	Favorable Market	Unfavorable Market	EMV	Minimum	Maximum
	0.7	0.3			
Big Plant	1000000	-600000	520000	-600000	1000000
Small Planmt	600000	-100000	390000	-100000	600000
No Plant	0	0	0	0	0
		Maximum->	520000	0	1000000
Column Best	1000000	0	700000	<- Expected Value Under Certainty	
			180000	<- EVPI	

- 1. Create an Excel spreadsheet showing the decision amounts for the maximax, maximin, and equally likely scenarios (decision under uncertainty)
- 2. Assuming that the favorable market is estimated at 70% and the unfavorable market at 30%, create an Excel spreadsheet for the decision amounts for a decision under risk (EMV); also calculate the EVPI
- 3. At what probability of a favorable market would the EMV of the Mighty Mack and Big Jack be equal

Equipment	Favorable Market ($)	Unfavorable Market ($)
The Mighty Mac	300,000	-200,000
The Big Jack	250,000	-100,000
The Oiler Otis	75,000	-18,000

- Do not look ahead !

Part 1 Solution

Alternatives	Favorable Market	Unfavorable Market	Maximax	Maximin	Equally Likely	
Mighty Mac	300000	-200000	300000	-18000	50000	75000
Big Jack	250000	-100000			75000	
Oiler Otis	75000	-18000			28500	

Part 1 Solution (formulas)

Alternatives	Favorable Market	Unfavorable Market	Maximax	Maximin	Equally Likely	
Mighty Mac	300000	-200000	=MAX(B5:B7)	=MAX(C5:C7)	=0.5*B5+0.5*C5	=MAX(F5:F7)
Big Jack	250000	-100000			=0.5*B6+0.5*C6	
Oiler Otis	75000	-18000			=0.5*B7+0.5*C7	

Part 2 Solution

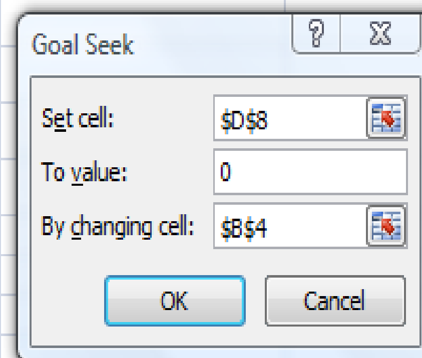
	A	B	C	D	E	F	G
1							
2							
3	**Alternatives**	**Favorable Market**	**Unfavorable Market**	**EMV**			
4		0.7	0.3				
5	**Mighty Mac**	300000	-200000	150000			
6	**Big Jack**	250000	-100000	145000			
7	**Oiler Otis**	75000	-18000	47100			
8			**Maximum**	150000			
9							
10	**Column Best**	300000	-18000	204600	**<- Expected value Under Certainty**		
11				54600	**<- EVPI**		

Part 2 Solution (formulas)

	A	B	C	D
1				
2				
3	**Alternatives**	**Favorable Market**	**Unfavorable Market**	**EMV**
4		0.7	=1-B4	
5	**Mighty Mac**	300000	-200000	=$B$4*B5+$C$4*C5
6	**Big Jack**	250000	-100000	=$B$4*B6+$C$4*C6
7	**Oiler Otis**	75000	-18000	=$B$4*B7+$C$4*C7
8			**Maximum**	=MAX(D5:D7)
9				
10	**Column Best**	=MAX(B5:B7)	=MAX(C5:C7)	=B4*B10+C4*C10
11				=D10-D8
12				

Part 3 Setup

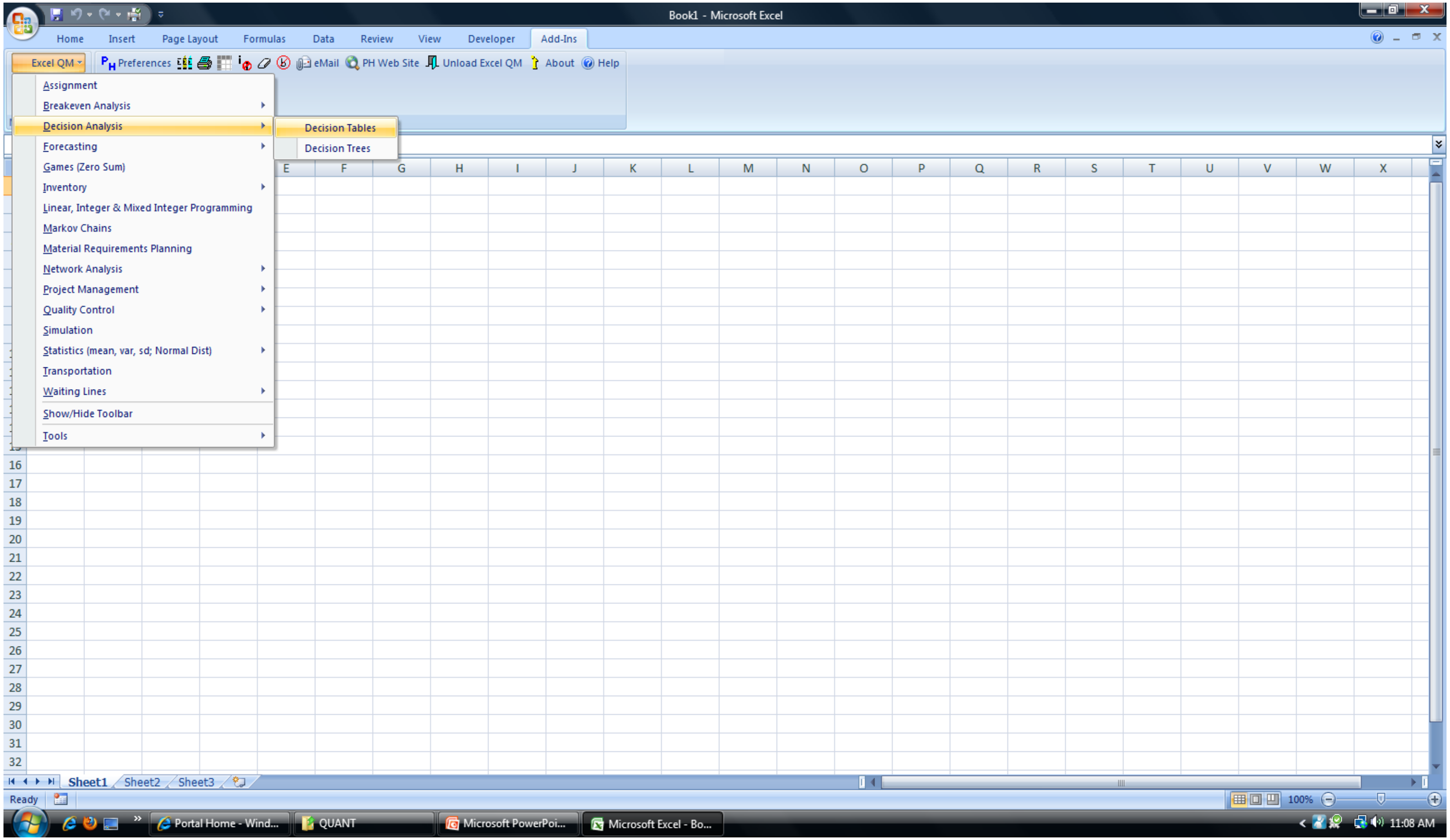
Alternatives	Favorable Market	Unfavorable Market	EMV
	0.7	0.3	
Mighty Mac	300000	-200000	150000
Big Jack	250000	-100000	145000
Oiler Otis	75000	-18000	
		Diference	5000



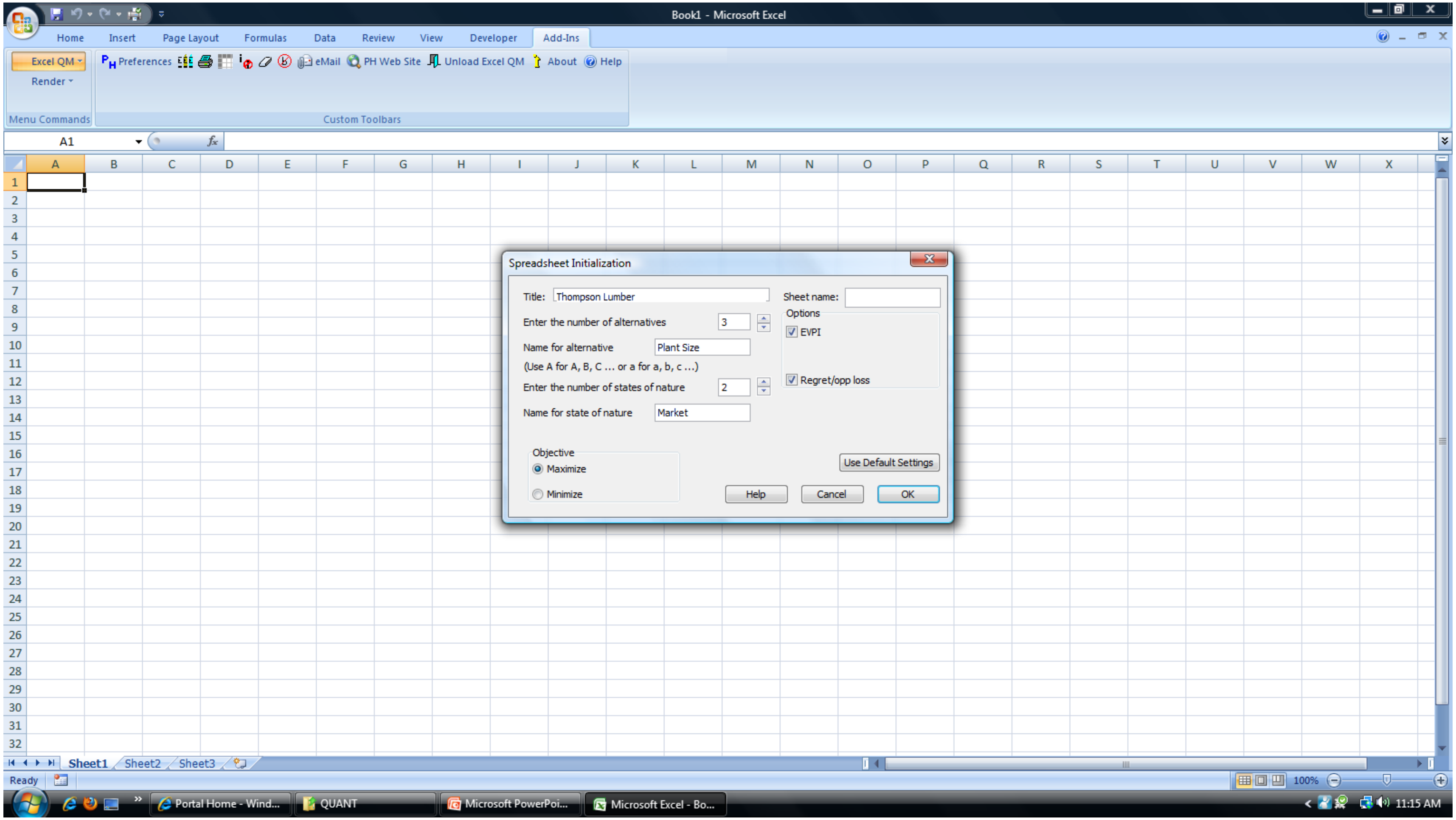
Part 3 Solution

	A	B	C	D
1				
2				
3	**Alternatives**	**Favorable Market**	**Unfavorable Market**	**EMV**
4		0.666666667	0.333333333	
5	**Mighty Mac**	300000	-200000	133333.333
6	**Big Jack**	250000	-100000	133333.333
7	**Oiler Otis**	75000	-18000	
8			**Diference**	0
9				

Using Excel QM to Solve Decision Theory Problems



Using Excel QM to Solve Decision Theory Problems (con't)



Using Excel QM to Solve Decision Theory Problems (con’t)

Compute the EMV for each alternative using the SUMPRODUCT function, the worst case using the MIN function, and the best case using the MAX function.

	A	B	C	D	E	F		
1	**Thompson Lumber**							
2								
3	**Decision Tables**							
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the first row if you want							
5	to compute the expected value.							
6	Data				**Results**			
7	Profit	Favorable Market	Unfavorable Market		EMV	Minimum	Maximum	Hurwicz
8	Probability	0.5	0.5					0.8
9	Large Plant	200000	-180000		=SUMPRODUCT(B$8:C$8,B9:C9)	=MIN(B9:C9)	=MAX(B9:C9)	=$I$8*G9+(1-$I$8)*F9
10	Small plant	100000	-20000		=SUMPRODUCT(B$8:C$8,B10:C10)	=MIN(B10:C10)	=MAX(B10:C10)	=$I$8*G10+(1-$I$8)*F10
11	Do nothing	0	0		=SUMPRODUCT(B$8:C$8,B11:C11)	=MIN(B11:C11)	=MAX(B11:C11)	
12					**=MAX(E9:E11)**	**=MAX(F9:F11)**	**=MAX(G9:G11)**	**=MAX(I9:I11)**
13								
14	**Expected Value of Perfect Information**							
15	Column best	=MAX(B9:B11)	=MAX(C9:C11		=SUMPRODUCT(B$8:C$8,B15:C15)	<-Expected value under certainty		
16					=E12	<-Best expected value		
17					=E15-E12	<-Expected value of perfect information		
18								
19	**Regret**							
20		=B7	=C7		Expected	Maximum		
21	=A8	=B8	=C8					
22	=A9	=B15 - B9	=C15 - C9		=SUMPRODUCT(B$8:C$8,B22:C22)	=MAX(B22:C22)		
23	=A10	=B15 - B10	=C15 - C10		=SUMPRODUCT(B$8:C$8,B23:C23)	=MAX(B23:C23)		
24	=A11	=B15 - B11	=C15 - C11		=SUMPRODUCT(B$8:C$8,B24:C24)	=MAX(B24:C24)		
25					**=MIN(E22:E24)**	**=MIN(F22:F24)**		
26								
27								
28								
29								

To calculate the EVPI, find the best outcome for each scenario.

Find the best outcome for each measure using the MAX function.

Use SUMPRODUCT to compute the product of the best outcomes by the probabilities and find the difference between this and the best expected value yielding the EVPI.

Using Excel QM to Solve Decision Theory Problems (con't)

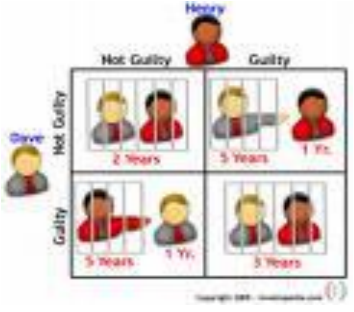
	A	B	C	D	E	F	G	H	I	J	K
1	**Thompson Lumber**										
2											
3	**Decision Tables**										
4		**Enter the profits or costs in the main body of the data table. Enter probabilities in the**									
5		**first row if you want to compute the expected value.**									
6	**Data**				**Results**						
7	Profit	Favorable Market	Unfavorable Market		EMV	Minimum	Maximum		Hurwicz		
8	Probability	0.5	0.5					coefficient	0.8		
9	Large Plant	200000	-180000		10000	-180000	200000		124000		
10	Small plant	100000	-20000		40000	-20000	100000		76000		
11	Do nothing	0	0		0	0	0				
12				**Maximum**	**40000**	**0**	**200000**		**124000**		
13											
14	**Expected Value of Perfect Information**										
15	Column best	200000	0		100000	<-Expected value under certainty					
16					40000	<-Best expected value					
17					60000	<-Expected value of perfect information					
18											
19	**Regret**										
20		Favorable M	Unfavorable Market		Expected	Maximum					
21	Probability	0.5	0.5								
22	Large Plant	0	180000		90000	180000					
23	Small plant	100000	20000		60000	100000					
24	Do nothing	200000	0		100000	200000					
25				**Minimum**	**60000**	**100000**					

Game Theory

- There are other quantitative methods for making decisions
- Game Theory was introduced in 1928 by John Von Neumann
- A game is any situation in which there are “rules of play” and “payoffs” for “outcomes”
- A two player “zero sum” game is one involving two parties where the amount won by one party is equal to the amount lost by the other party
- For management decisions, the two players are the manager and nature

Game Theory (con’t)

- Like decision trees there are games involving perfect information, under risk, and under uncertainty
- For example chess and checkers are games of perfect information since the whole board is visible; poker or bridge are games of under risk since not all information is visible
- Algorithms for game theory are like those for playing chess

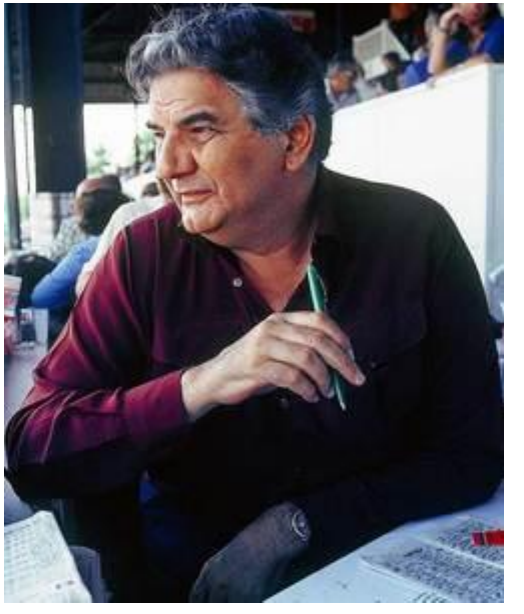


Determining Probabilities

- There are several ways to determine probabilities:
  - Objectively:
    - Classical (logical) – counting possibilities
    - Observance – past occurrences are statistically tabulated to calculate probabilities (frequencies)
  - Subjective:
    - Expert opinion
    - Odds Forecasting

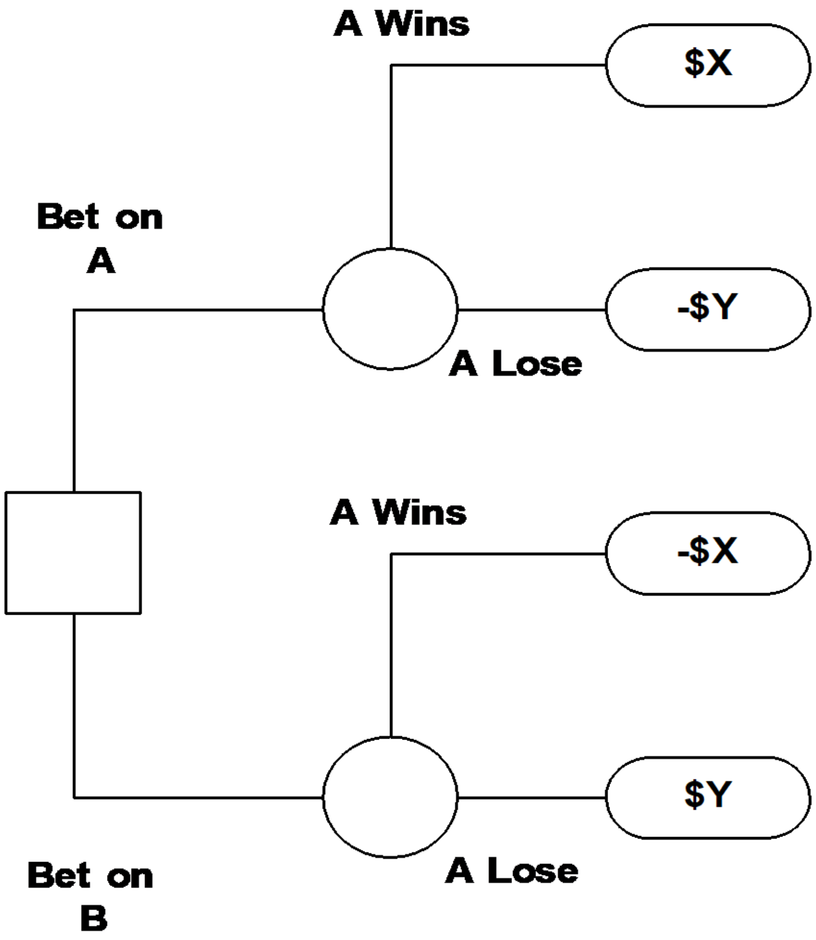
Odds (making) Forecasting

- This involves finding the point (usually expressed in a dollar bet) that a decision maker <u>is willing to accept either side of the bet</u>
- In sports wagering, an “oddsmaker” is indifferent to which team you bet on because he has set the odds to the point at which he is indifferent



Decision Tree for Odds Making

[set X and Y for indifference, EMV is same for either decision]



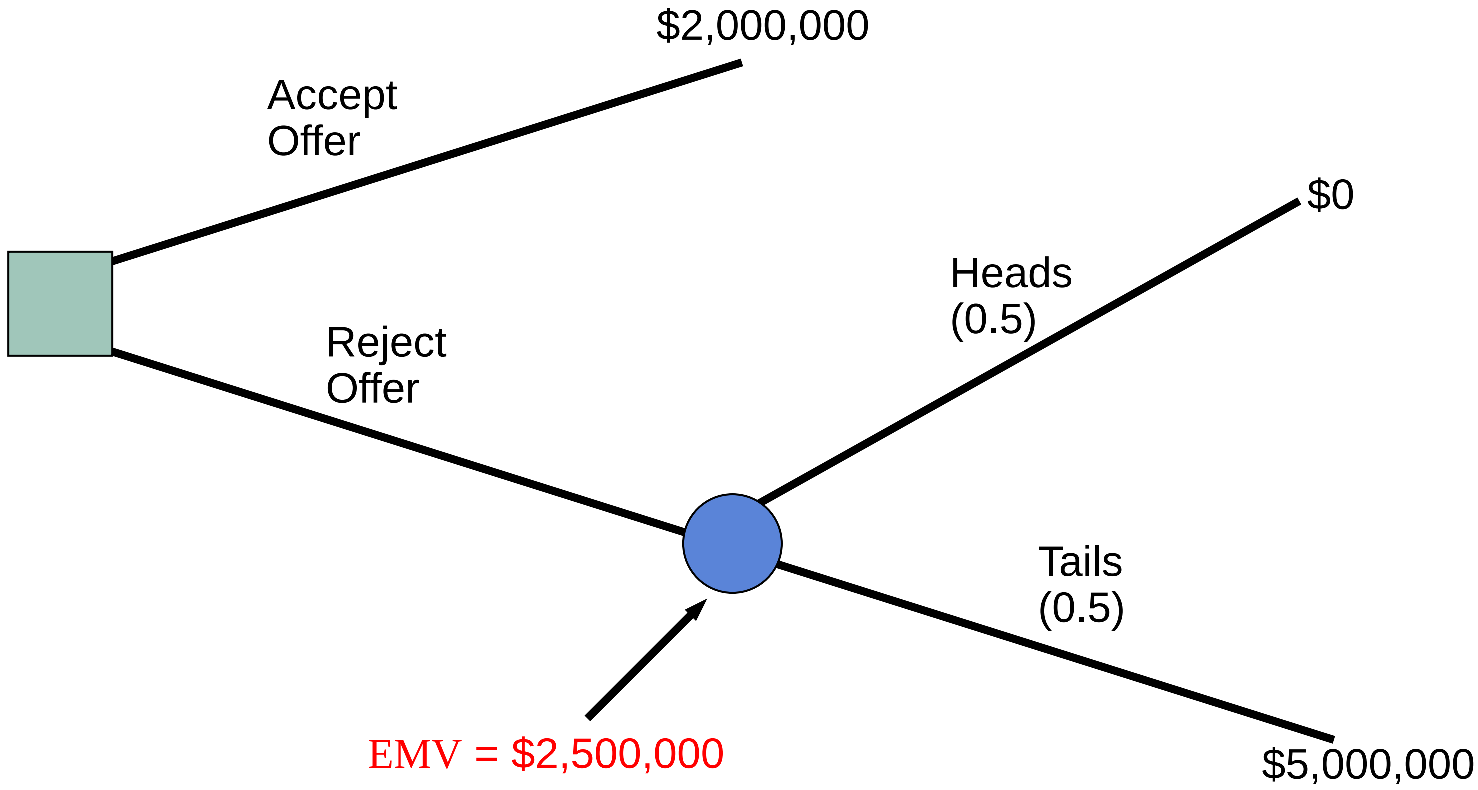
Odds (making) Forecasting (con't)

- For indifference we will have equal EMV's:
  - X*P(A Wins) – Y*[1-P(A Wins)] = Y*[1-P(A Wins)] – X*P(A Wins)
- Reduction leads to:
  - (X + Y) * P(A Wins) – Y = 0
- Solving for P(A Wins):
  - P(A Wins) = Y/(Y + X)
- For example, if the oddsmaker is willing to win $100 if A wins and loss $225 if A loses:
  - P(A Wins) = (225/(325) = 0.7 [or 7 to 3 for A]

Utility Theory

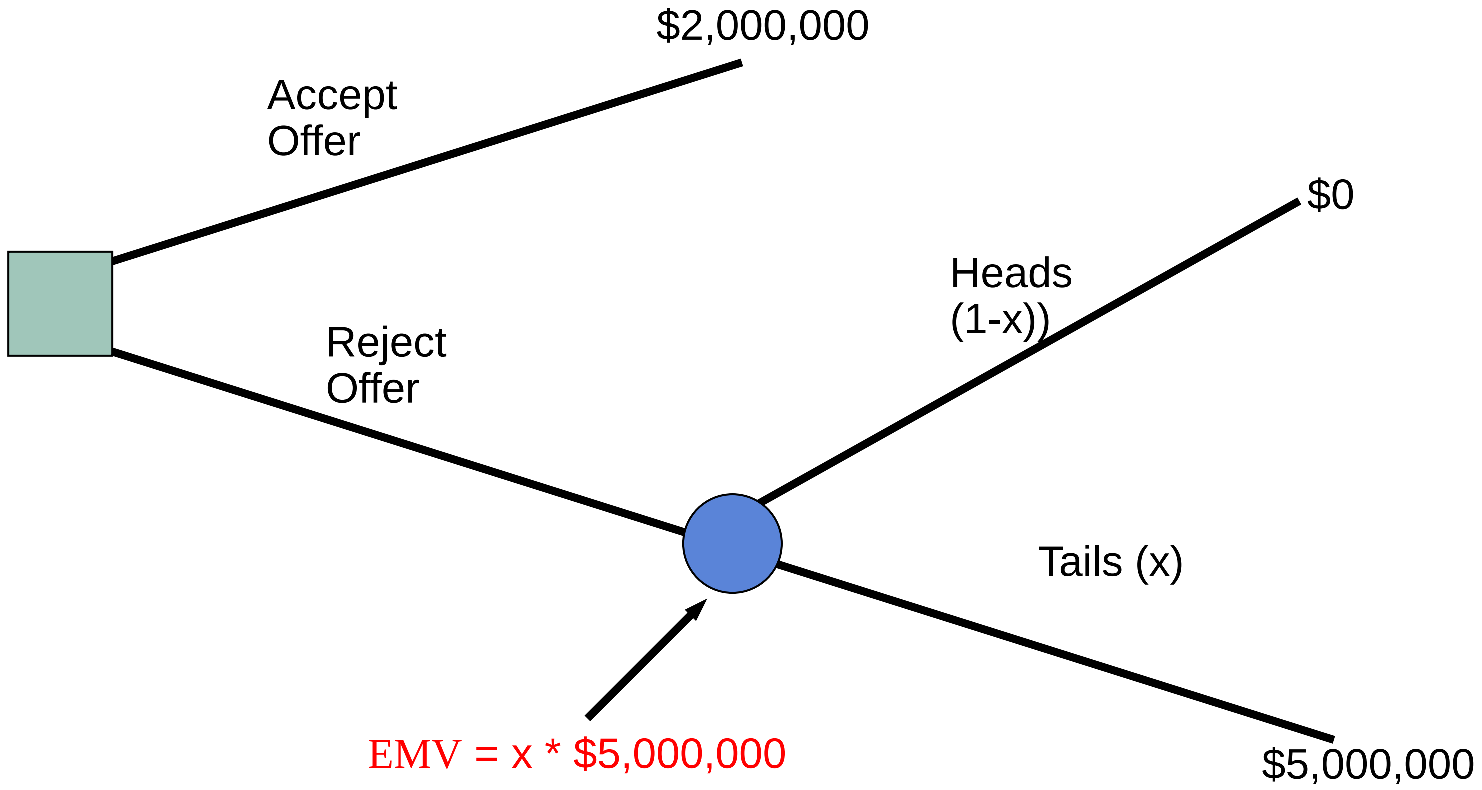
- Monetary value is not always a true indicator of the overall value of the result of a decision
- The overall value of a decision is called *utility*
- Rational people may make decisions to maximize their utility

I’ll give you $ 2 million or you can flip a coin to see if you get $ 5 million or nothing; which will you do ?



Even though the EMV is higher for the choice to flip (accept offer), one may choose not to flip due to the fact that the “utility” of being “sure” is better for them.

What value of x (between zero and 1) would make you want to flip, instead of taking the sure $2 million ?



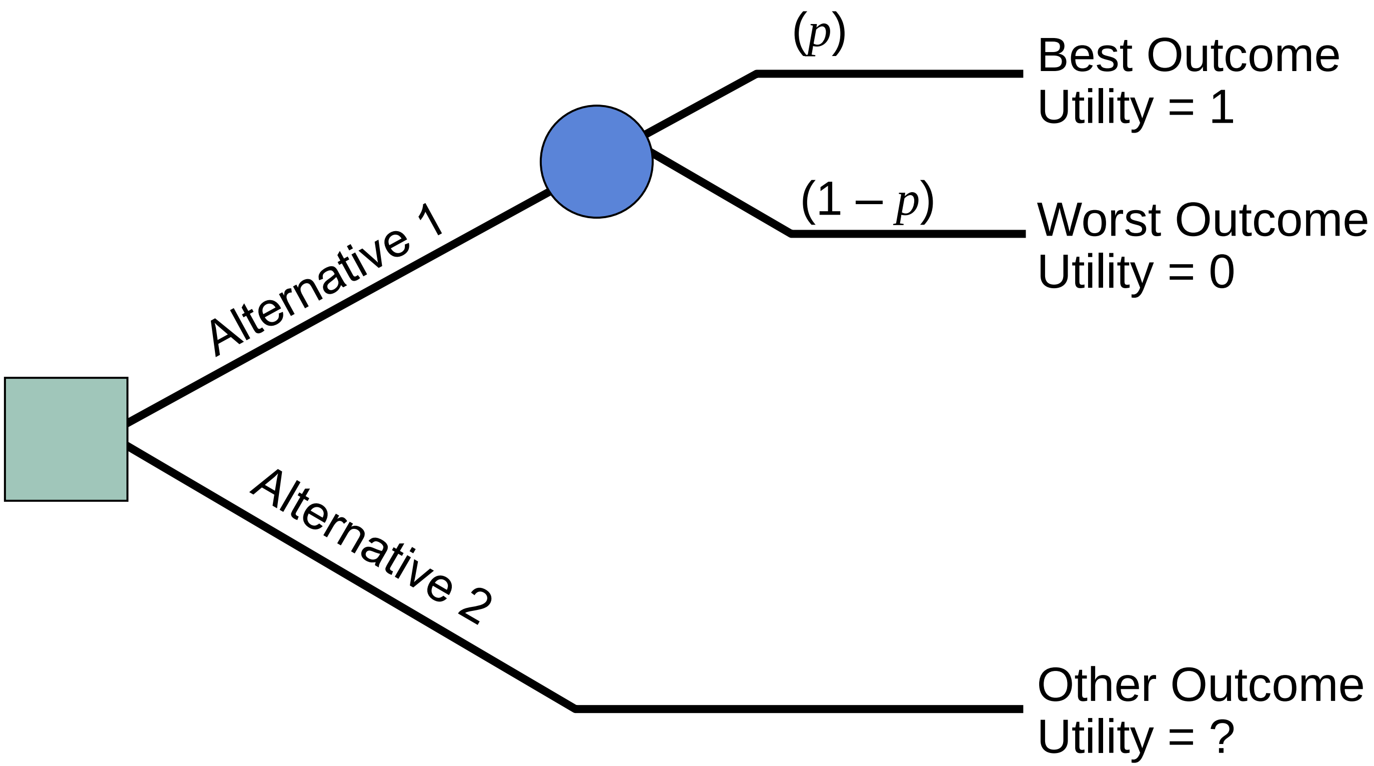
Utility Assessment

- *Utility assessment* assigns the worst outcome a utility of 0, and the best outcome, a utility of 1; other outcomes have utilities between 0 and 1
- A *standard gamble* is used to determine utility values
- When you are indifferent, the utility values are equal:

Expected utility of alternative 2 = Expected utility of alternative 1

Standard Gamble

(what value of p makes you indifferent ?)

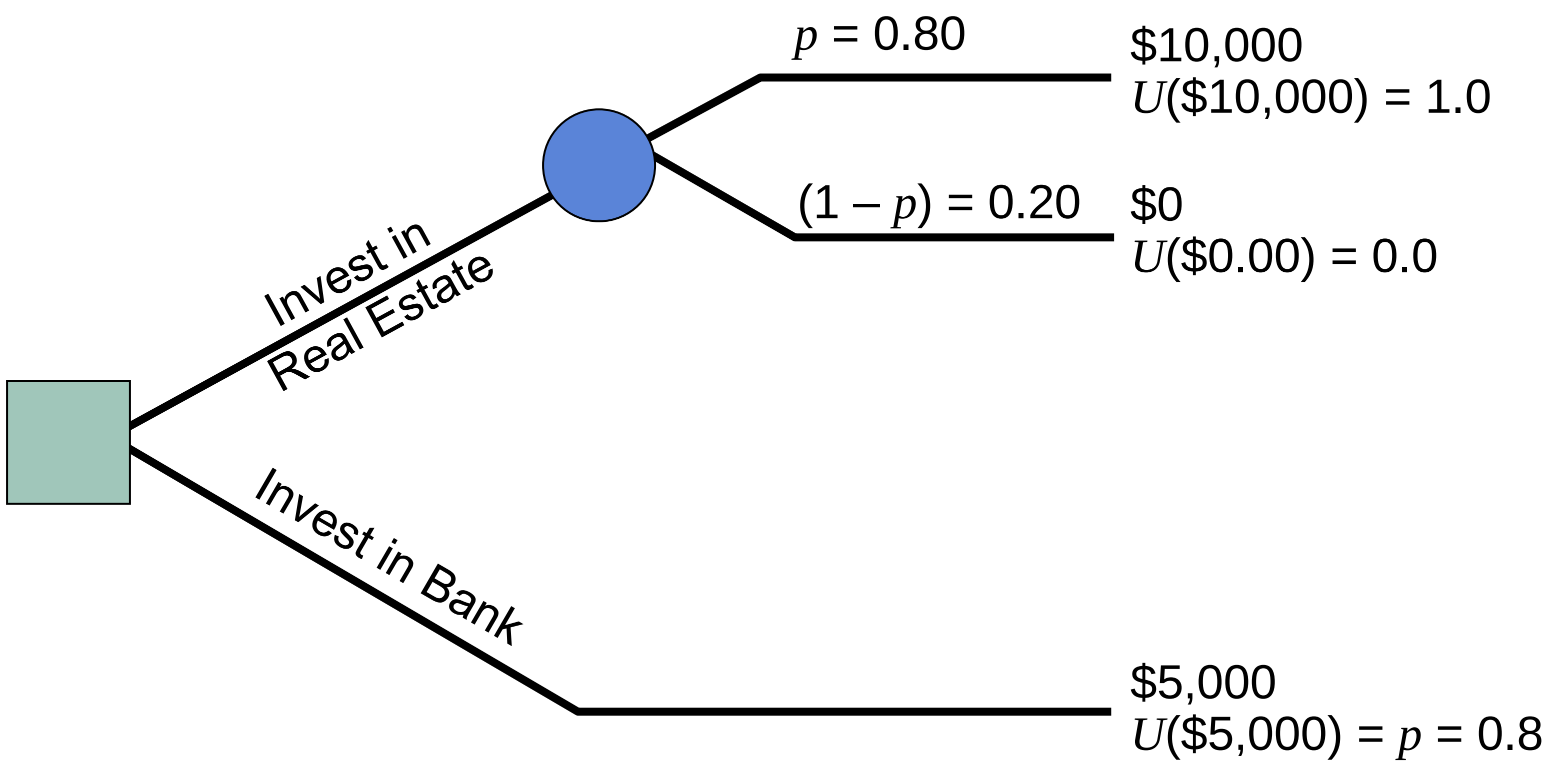


Where p is the probability of best outcome.

Investment Example

- Jane Dickson wants to construct a utility curve revealing her preference for money between $0 and $10,000
- A utility curve plots the utility value versus the monetary value
- An investment in a bank will result in $5,000
- An investment in real estate will result in $0 or $10,000
- A pure EMV choice would set p at 0.5
- However for Jane, unless there is an 80% chance of getting $10,000 from the real estate deal, she would prefer to have her money in the bank
- So if p = 0.80, Jane is indifferent between the bank or the real estate investment

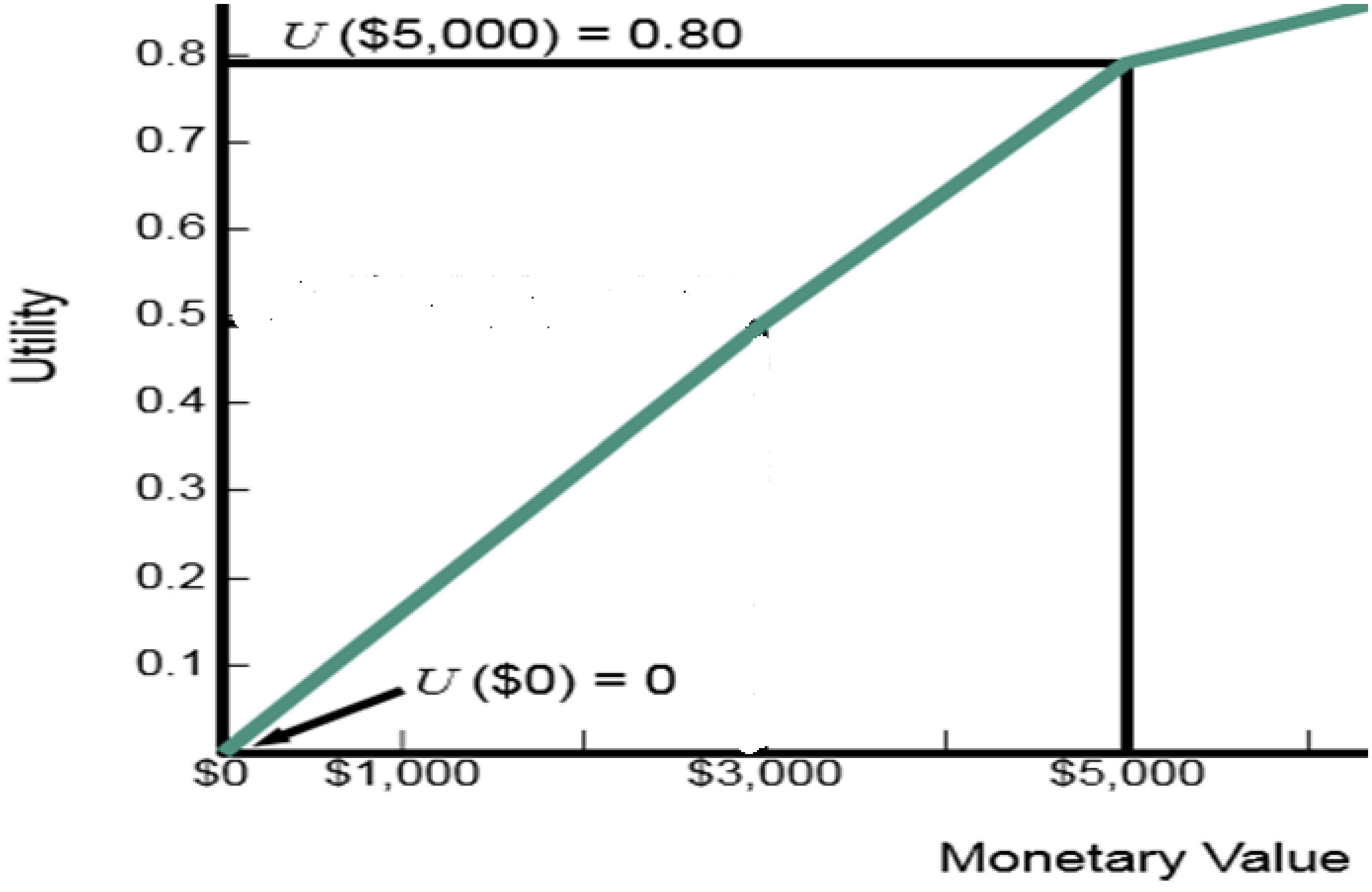
Investment Example (con't)



Utility for \$5,000 = $U(\$5{,}000) = pU(\$10{,}000) + (1 - p)U(\$0)$

$= (0.8)(1) + (0.2)(0) = 0.8$

Utility for $5000 is 0.8



Investment Example

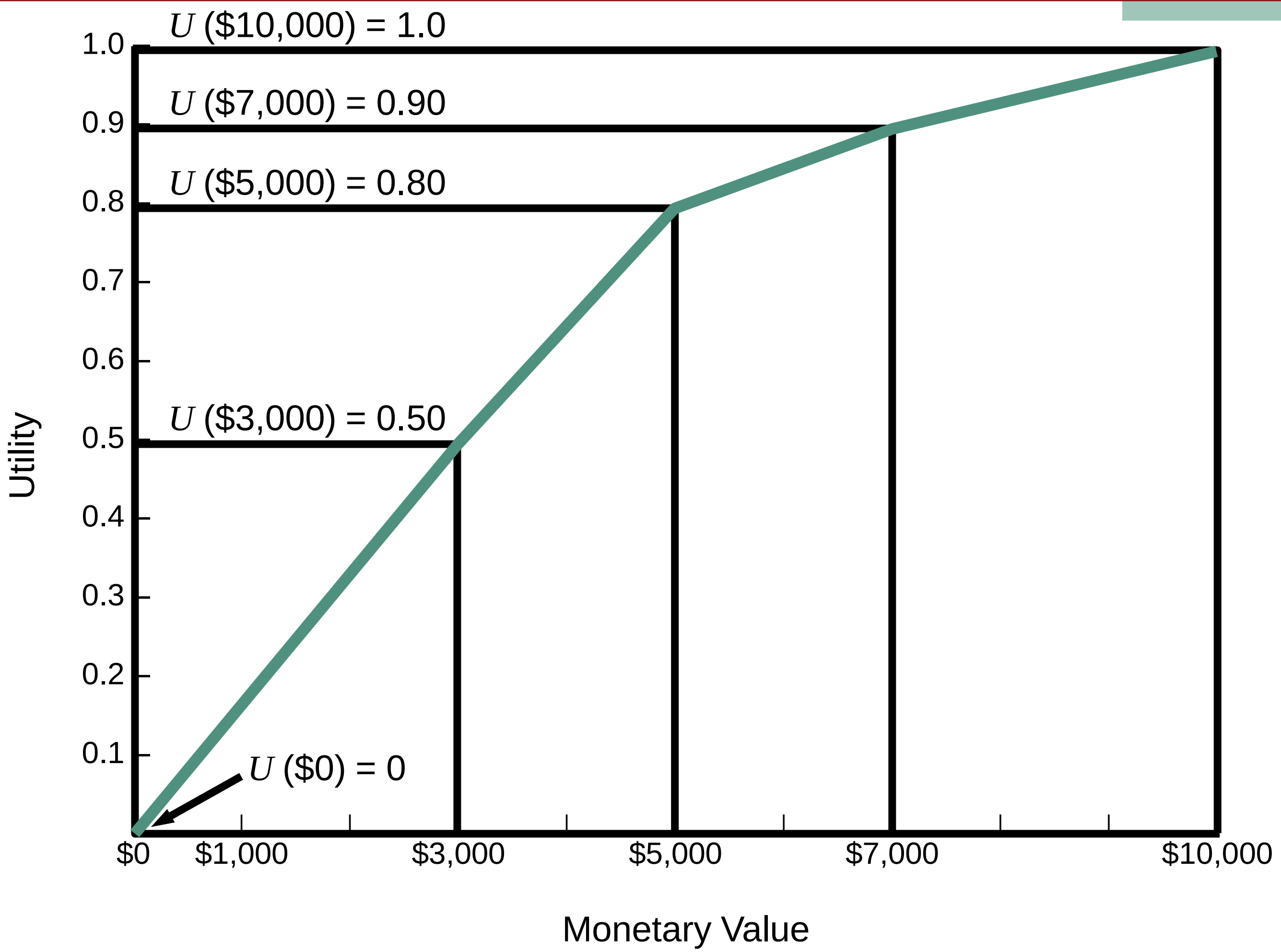
- We can assess other utility values in the same way
- For Jane these are:

Utility for $7,000 = 0.90

Utility for $3,000 = 0.50

- Using the three utilities for different dollar amounts, she can construct a utility curve

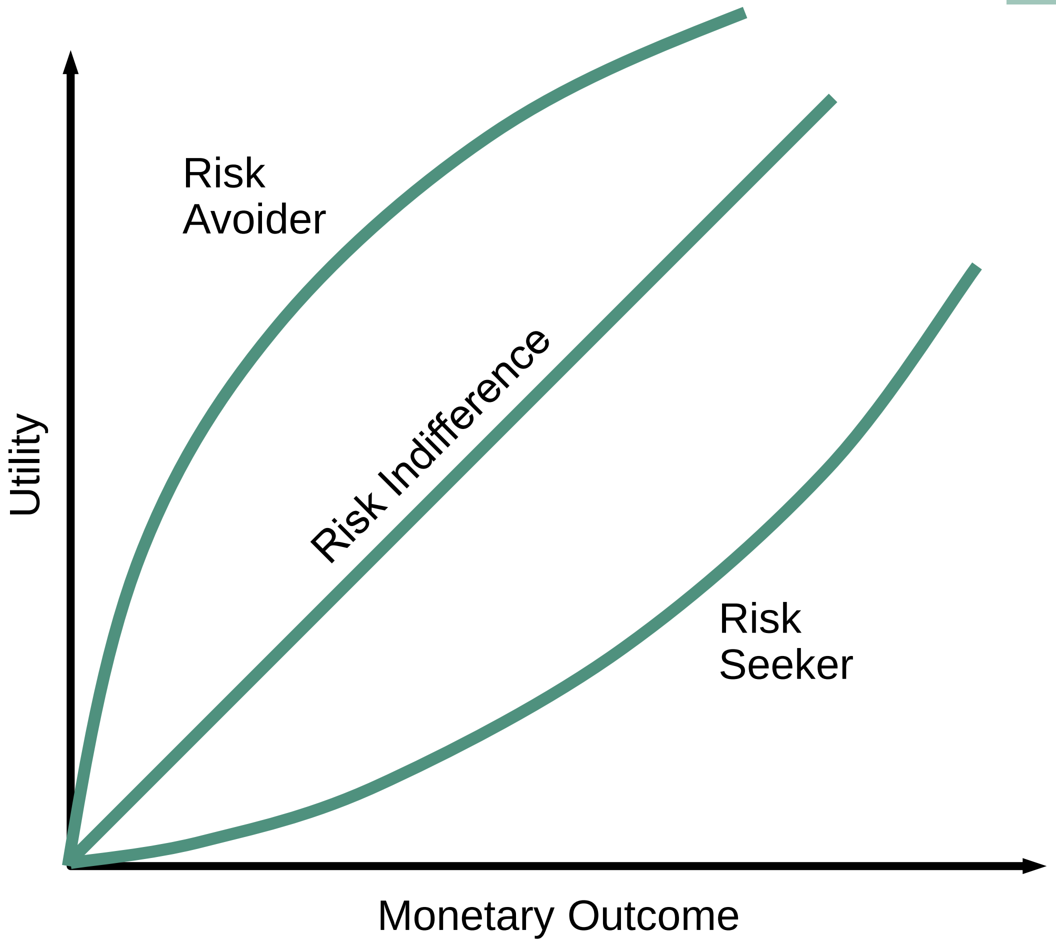
Utility Curve



Utility Curve

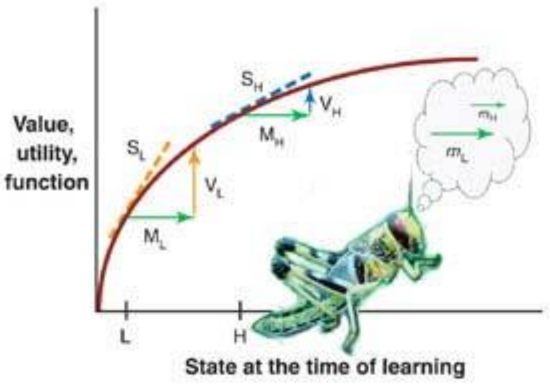
- Jane's utility curve is typical of a risk avoider
  - A risk avoider gets less utility from greater risk
    - Avoids situations where high losses might occur
    - As monetary value increases, the utility curve increases at a slower rate
  - A risk seeker gets more utility from greater risk
    - As monetary value increases, the utility curve increases at a faster rate
    - Someone who is indifferent will have a linear utility curve

Utility Curve



Utility as a Decision-Making Criteria

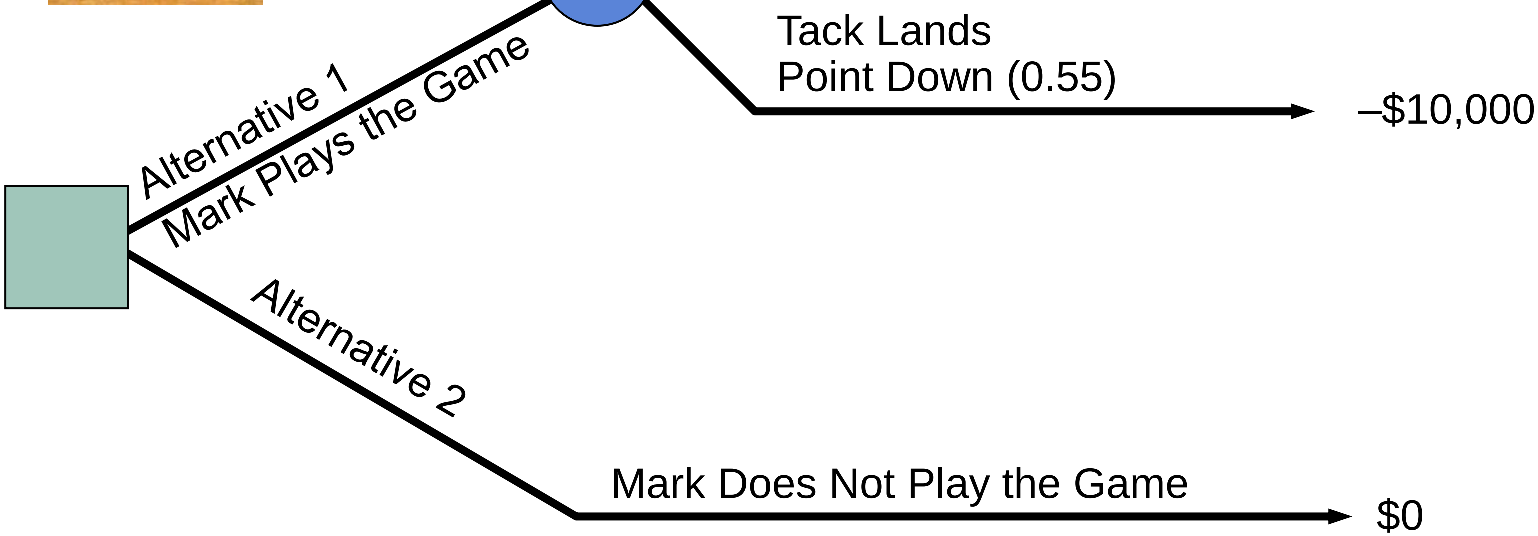
- Once a utility curve has been developed it can be used in making decisions
- Replace monetary outcomes with utility values
- The expected utility is computed instead of the EMV



Utility as a Decision-Making Criteria (con't)

- Mark Simkin loves to gamble, and he has $20000 to gamble
- He plays a game tossing thumbtacks in the air
- If the thumbtack lands point up, Mark wins $10,000
- If the thumbtack lands point down, Mark loses $10,000
- Should Mark play the game (alternative 1)?

Utility as a Decision-Making Criteria (con’t)



Utility as a Decision-Making Criteria (con't)

- Step 1– Define Mark's utilities (next slide)

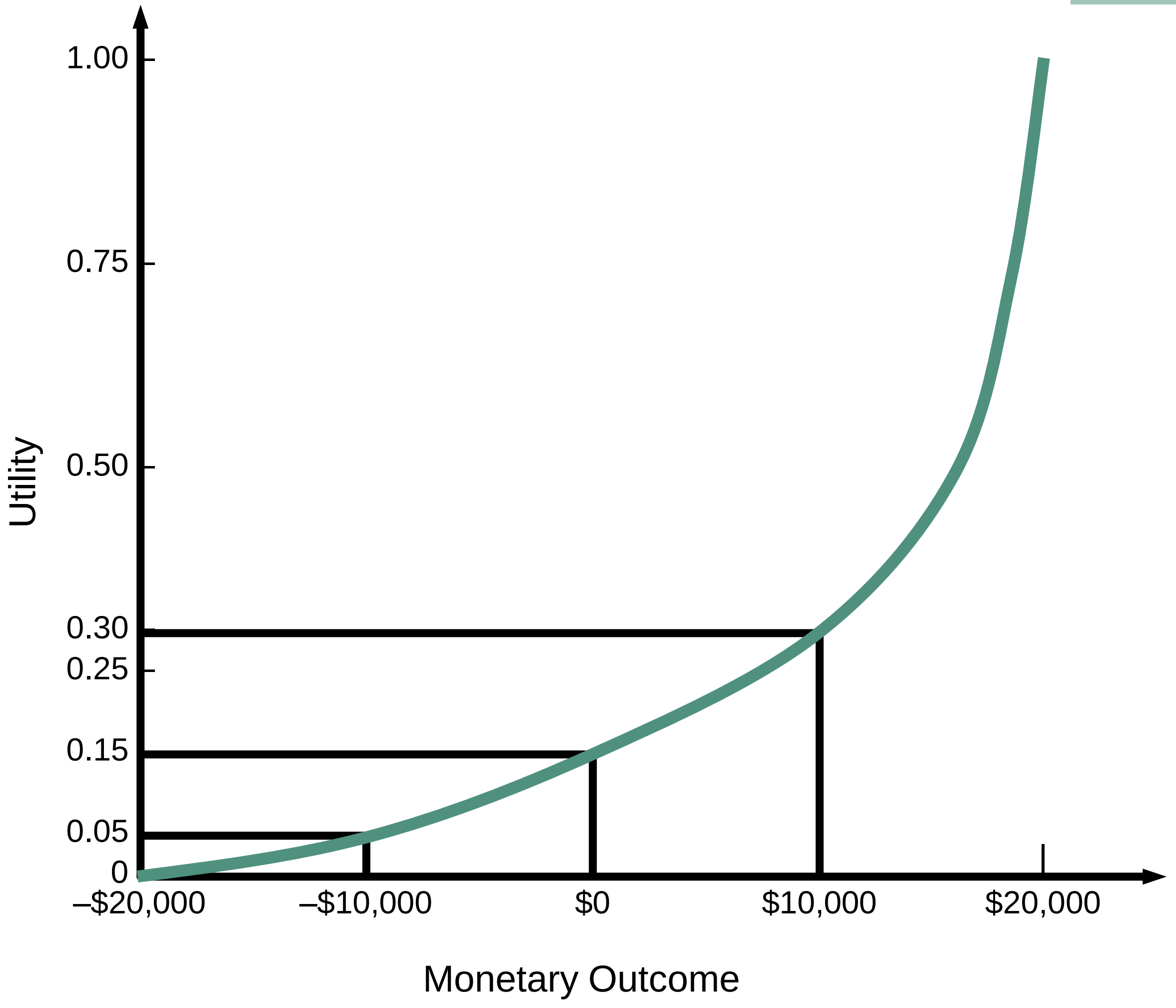
$$U(–\$10,000) = 0.05$$
$$U(\$0) = 0.15$$
$$U(\$10,000) = 0.30$$

- Step 2 – Replace monetary values with utility values (slide after next)

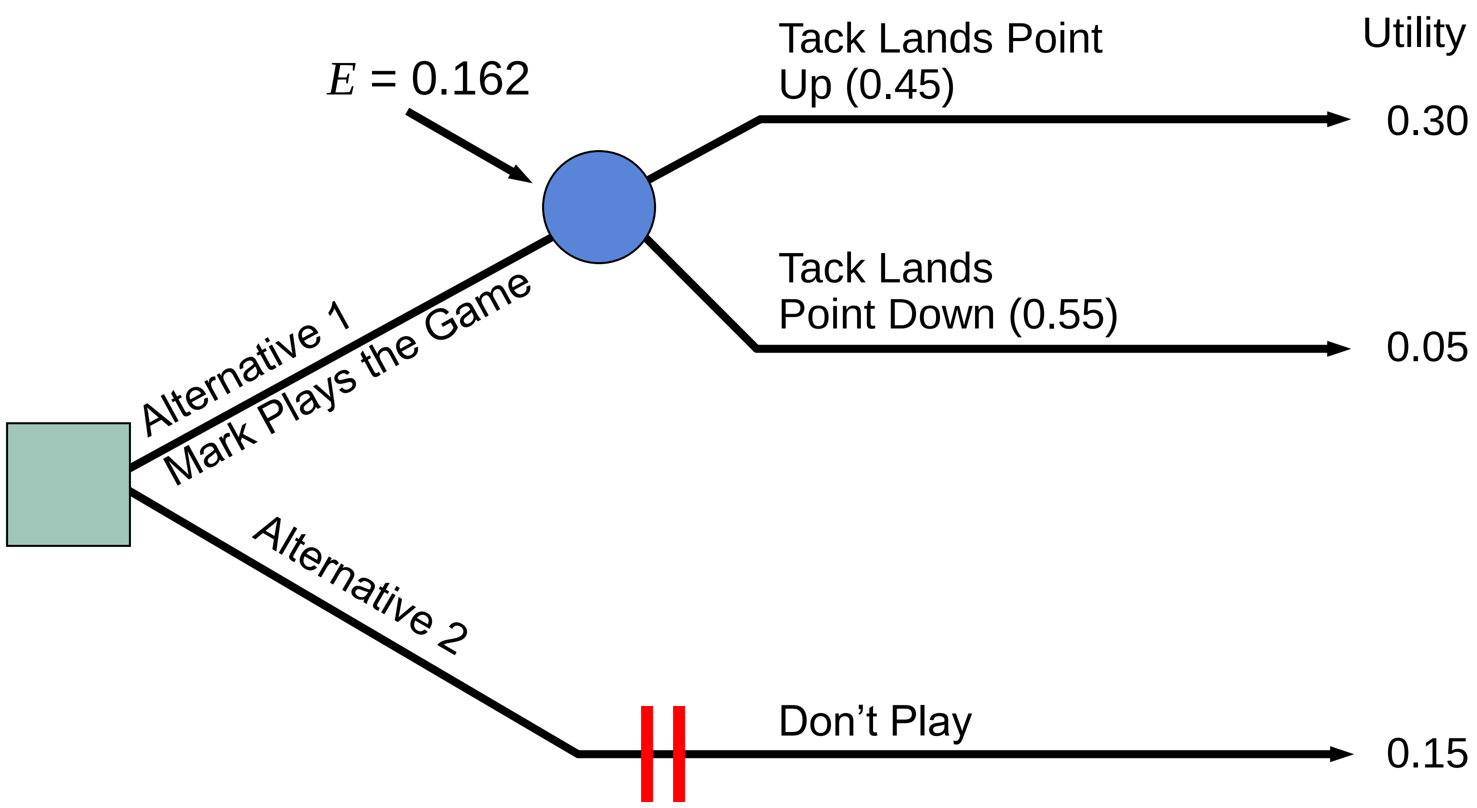
E(alternative 1: play the game) = (0.45)(0.30) + (0.55)(0.05)
= 0.135 + 0.027 = 0.162

E(alternative 2: don't play the game) = 0.15

Mark’s Utility Curve



Utility as a Decision-Making Criteria (con't)

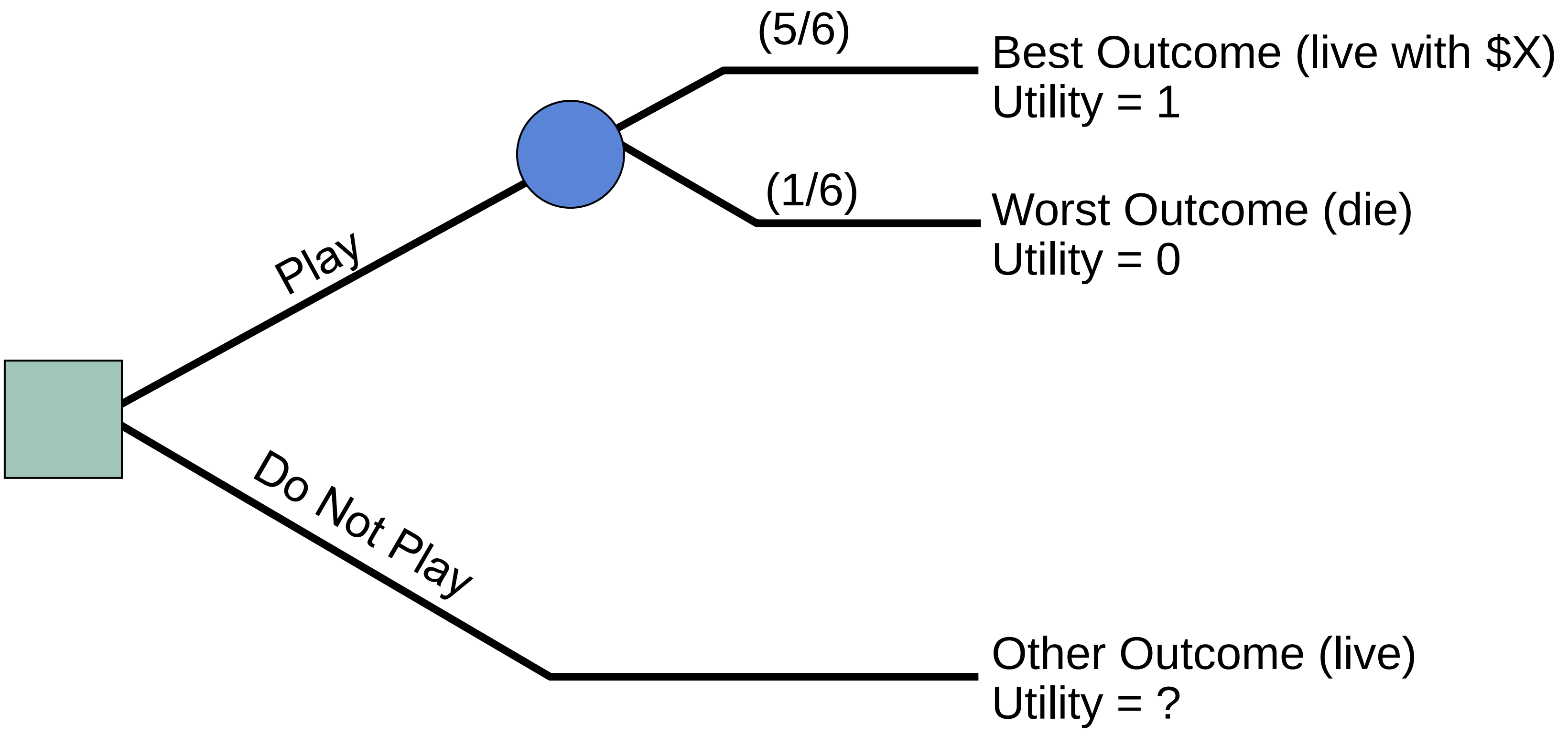


So Mark would chose to play the game, since that gives him greater utility !

Russian Roulette

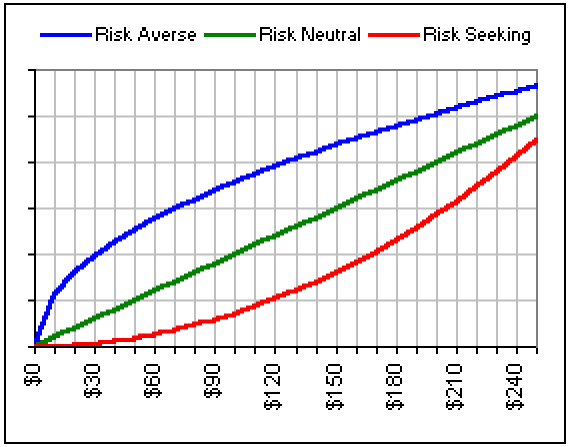
- Do not play – live
- Play
  - Die (1/6 probability)
  - or
  - X amount of money
- **What value of X would make you play ?**

Russian Roulette



Utility Lab

- You can invest $100,000 in a bank CD paying 5%
- Or you can invest in the stock market where:
  - You can lose all your money
  - Or you can earn 10%
- [Construct you own utility curve]()



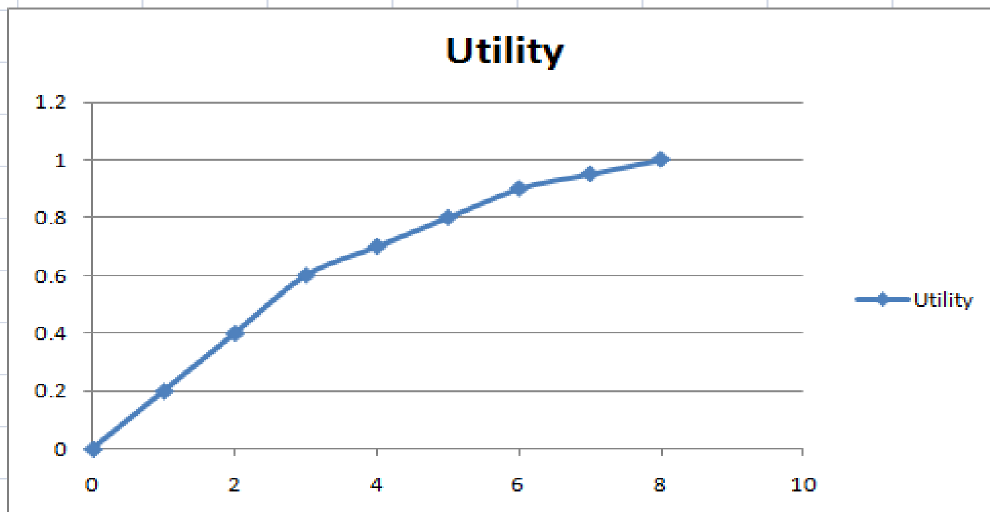
Hint

- Let p be the probability of earning the 10% in the market
- What value of p would make you indifferent ?
  - That is the utility of 5% (the CD rate)
- Now do the same analysis for the utility of bank CD rates of 1%, 2%, 3%, 4%, 6%, and 7%
- Now plot the bank CD rate on the X axis, and the utility of that rate on the Y axis (using Excel)

- Do not look ahead !

Utility Function for Earning Rates

CD Rate	Utility
0	0
1	0.2
2	0.4
3	0.6
4	0.7
5	0.8
6	0.9
7	0.95
8	1



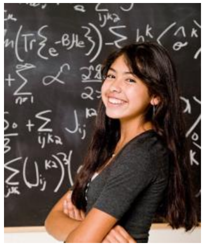
References

- Theory of Decision under Uncertainty (Econometric Society Monographs) by Itzhak Gilboa
- Choices: An Introduction to Decision Theory by Michael D. Resnik
- Statistical Decision Theory and Bayesian Analysis (Springer Series in Statistics) by James O. Berger
- Decision Analysis, Game Theory, and Information (University Casebook Series) by Louis Kaplow and Steven Shavell
- Decision Theory: Principles and Approaches (Wiley Series in Probability and Statistics) by Giovanni Parmigiani and Lurdes Inoue
- Introduction to Statistical Decision Theory (Global Environmental Accord: S) by John W. Pratt, Howard Raiffa, and Robert Schlaifer

Homework

- Textbook Chapter 2 and 3
- Quiz on these slides and Chapter 3 next session
- Complete your risk utility graph
- Questions ("Discussion Questions and Problems") to be answered: 2, 3, 4, 5, 9, and 13 from Chapter 3
- Project One →

Project 1



- Amy Austin is considering going back to school at nights
- She will either get a masters degree in Accounting (her first degree is in Accounting) or an MBA
- She has calculated the net present value (NPV) of getting each of these degrees based upon the cost of each degree and the increase in salary she would get over her 20 years of remaining career

Project (con't)

- She knows that she will either stay as an accounting professional or will move into upper management
- She estimates that her probability of staying an accounting professional is 55%
- The NPV of an MBA is 50,000 as an accounting professional and 350,000 in upper management
- The NPV of the masters in Accounting degree is 180,000 as an accounting professional and 280,000 in upper management

Project (con't)

- Which degree should she pursue ?
- Perform a decision analysis (both graphically and in Excel) and determine:
  - Her conservative choice
  - Her risky choice
  - Her EMV choice
- Also determine her EVPI